

Unemployment and the Labor Share[☆]

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Abstract

The labor share fluctuates over the business cycle. To explain this behavior, we develop a novel model featuring direct competition between heterogeneous firms to hire workers. This simultaneously endogenizes both average match productivity and the division of output between workers and firms. In existing matches, wages partly reflect labor market conditions at the time of hiring. A positive TFP shock therefore reduces the aggregate labor share, making it counter-cyclical. However, greater competition and lower unemployment increase labor's share among new firms. As more firms enter, the aggregate labor share rises and eventually overshoots its initial level, as in the data.

Keywords: Labor share; Factor shares; Unemployment; Search; Heterogeneous firms

JEL Classification: E23, E24, E25, E32, J64

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1. Introduction

The relative constancy of the labor share has for decades been seen as a stylized fact in macroeconomics, as evidenced by the widespread usage of the Cobb-Douglas aggregate production function. Recently, however, the labor share has been shown to fluctuate in a cyclical manner. In particular, Rios-Rull and Santaaulalia-Llopis (2010) show that the labor share declines in response to a positive TFP shock – making it *counter-cyclical* – but this drop quickly reverts and the labor share rises over time, eventually *overshooting* its initial level. This paper aims to understand the mechanism behind such cyclical movements of the labor share.

Building on the basic framework of Mangin (2017), we develop a dynamic search and matching model in which firms with heterogeneous productivities compete for workers. In contrast to much of the existing literature, which assumes a particular aggregate production function, the model simultaneously endogenizes both wages and average match output. We incorporate both aggregate TFP and investment-specific technology shocks and find that the framework yields rich yet analytically tractable labor market dynamics that are consistent with the data. In particular, we show that the model can account for both the counter-cyclicality of the labor share *and* its overshooting behavior following an aggregate TFP shock.

Unlike Diamond-Mortensen-Pissarides (DMP) models with bilateral (one-on-one) meetings, a key feature of the model is that a worker may be simultaneously approached by two or more competing firms that directly compete to hire the worker. In such multilateral (many-on-one) meetings, workers are hired by the most productive firm. This endogenizes the average match output, or labor productivity, which is increasing in the market tightness since workers can be more selective when there are more competing firms per worker. At the same time, wages also endogenously reflect the degree of competition at the time of hiring. Specifically, wages in multilateral meetings are equal to the productivity of the second most productive firm competing for a worker. Therefore, in our model, production and wage determination are intimately linked. This contrasts with DMP models with Nash bargaining, which rely on using an exogenous bargaining parameter that is unrelated to the production technology.

We assume a Pareto distribution for firm productivity levels, which enables us to analytically characterize the model’s dynamics, including those of the aggregate labor share. In steady state, the labor share depends on the degree of competition for workers, the reservation wage, the level of TFP, and the dispersion of firm productivity levels. In particular, we show that the steady state labor share is increasing in the level of TFP provided that the reservation wage is sufficiently responsive – a condition that is satisfied under our calibration. The mechanism is intuitive: higher TFP leads to greater firm entry and lower unemployment, thereby increasing the labor share through stronger competition to hire workers and a higher reservation wage.

This steady state result suggests a positive relationship between aggregate TFP and the labor share. However, we know that the labor share is *counter-cyclical* in the data. In fact, the full dynamic model can account for both the counter-cyclicality and the overshooting of the labor share. This highlights the importance of studying its dynamics. To understand these dynamics, we need to consider the cohort effects generated by firm competition for workers.

Outside the steady state, matches of various “vintages” coexist in every period. Since both wages and match productivity are determined through the process of direct competition to hire workers, both are influenced by the labor market conditions prevailing *at the time of hiring*. As a result, the model generates cohort effects that lead to rich dynamics for the labor share. In particular, the aggregate labor share is influenced by the composition of the employment pool with respect to match vintages (i.e. labor market conditions at the time of hiring). The systematic variation in the composition of matches of various vintages is key for understanding the dynamics of the aggregate labor share.

In a calibrated version of the model, we show that the aggregate labor share falls in response to a positive TFP shock: the labor share is counter-cyclical. This happens because wages of existing workers are partially rigid, since they reflect the degree of competition at the time of hiring. However, following a positive TFP shock, the degree of competition between firms strengthens and unemployment falls because it is now more attractive to hire workers. This, in turn, endogenously increases the labor share of newly hired workers. Over time, as the composition of matches shifts towards vintages formed in periods of stronger competition, the aggregate labor share rises and

1 eventually reaches a level higher than its initial one: the labor share overshoots.

2 We also investigate to what extent our model is able to explain the observed time path of the
3 U.S. labor share and how important are the novel features of our model – namely, the competition
4 and cohort effects – for achieving this. To do this, we use the calibrated model and *estimate*
5 the underlying aggregate shocks using data on real GDP and unemployment. We show that the
6 resulting model-predicted time series for the labor share tracks the data very closely. Importantly,
7 we conduct the same exercise in a variant of our model, which retains the same equilibrium
8 conditions but in which we “switch off” the competition and cohort effects. The resulting time
9 series for the labor share is essentially uncorrelated with its counterpart in the data. Therefore,
10 the model’s key features – competition and cohort effects – are crucial for understanding the time
11 series pattern of the labor share that we observe in the data.

12 Finally, in addition to accounting for the dynamics of the labor share – the key focus of
13 this paper – we show that the calibrated model also performs well in replicating business cycle
14 patterns of other labor market variables. Moreover, the model offers a set of new predictions
15 that are absent in standard search and matching models. While of independent interest, these
16 also serve as supporting evidence for the underlying mechanism. First, the model predicts that
17 the wage elasticity with respect to aggregate labor productivity is larger for new hires than for
18 existing workers, which is consistent with the data (see e.g. Bils, 1985; Haefke et al., 2013; Gertler
19 et al., 2016). Second, the model predicts that labor productivity features an overshooting pattern
20 in response to investment-specific shocks, as observed in the data (see e.g. Fisher, 2006; Canova
21 et al., 2013). Finally, a key prediction of the model is the presence of firm and worker cohort
22 effects – driven by aggregate conditions at the time of hiring – which have also been documented
23 in the data (see e.g. Oreopoulos et al., 2012; Sedláček and Sterk, 2017).

24 There has been a resurgence of interest in the labor share due to its recent trend decline both
25 in the U.S. and globally. In particular, King and Watson (2012) document a sharp fall in the
26 U.S. labor share during the 2000s. Elsby et al. (2013) discuss this recent decline and identify
27 globalization and offshoring as potentially important factors. Karabarbounis and Neiman (2014)
28 document a global decline in the corporate labor share since the early 1980s and attribute this to
29 a fall in the relative price of investment goods. Piketty (2014) and Piketty and Zucman (2014)
30 suggest the labor share decline arises from capital accumulation. This literature follows earlier
31 papers on medium-run changes in the labor share, including Blanchard (1997), who examines the
32 effects of labor market deregulation and changes in the degree of unionization, and Bentolila and
33 Saint-Paul (2003), who consider the effects on the labor share of changes in workers’ bargaining
34 power, among other factors.

35 In contrast to the above studies, this paper focuses on the *cyclical* fluctuations of the labor
36 share. We do so by extending the static search and matching framework of Mangin (2017) to a
37 dynamic environment with aggregate shocks. The cyclical nature of the labor share has received
38 relatively less attention in the literature. In an earlier contribution, Gomme and Greenwood
39 (1995) argue that optimal labor contracting between entrepreneurs and workers can explain the
40 counter-cyclical behavior of labor’s share at a quarterly frequency. Recently, Choi and Rios-Rull
41 (2009) and Colciago and Rossi (2015) suggest that, in addition to non-competitive wages, deeper
42 insight into the nature of the production technology may be necessary to understand both the
43 counter-cyclical and the overshooting of the labor share. While the former study a search and
44 matching model with a C.E.S. production function, the latter consider a model with counter-
45 cyclical markups.

46 The remainder of this paper is structured as follows. Section 2 presents the model and de-
47 rives expressions for the dynamics of unemployment, output, and the labor share. It provides a
48 description of the wage determination mechanism and expressions for wages and factor shares for
49 arbitrary distributions of firm productivities, before specializing to the Pareto distribution. Sec-
50 tion 3 establishes the existence and uniqueness of the steady state equilibrium and derives some
51 comparative statics results. In Section 4, we calibrate the model and present the quantitative
52 results and analysis. Section 5 concludes. All proofs are found in the Appendix.¹

¹The Appendix is available online on the journal’s website.

2. Model

There is an infinite number of discrete time periods. In each period, there is a continuum of homogeneous risk-neutral workers of measure L . At the start of period t , there is a continuum of measure U_t of unemployed workers and a continuum of risk-neutral potential firms. The measure of *entering* firms is V_t and the ratio of entering firms to unemployed workers is $\phi_t = V_t/U_t$. Both workers and firms discount future payoffs using a discount factor $\beta \in (0, 1)$.

To enter, potential firms must pay a cost C_t , the cost of purchasing one unit of capital. We interpret exogenous shocks to C_t as *investment-specific technology shocks*. After paying the cost C_t , all firms search for a worker.

Firms approach unemployed workers at random. For the sake of tractability, workers who are already employed cannot be targeted by firms: there is no on-the-job search. After meeting with a worker, firms privately draw an idiosyncratic permanent match-specific productivity x from a distribution with cdf $G(x)$.²

Assumption 1. *The underlying distribution of firm productivities has cdf $G(x)$, pdf $g(x)$, support $[x_{\min}, \infty)$ where $x_{\min} \geq 0$, a finite mean, and no mass points.*

Not all matches may be acceptable for both workers and firms: only matches with permanent productivity $x \geq x_t^c$ for some cut-off productivity x_t^c are accepted by both workers and firms in period t . Firms with productivity $x \geq x_t^c$ will compete to hire workers; firms with productivity $x \leq x_t^c$ exit immediately.³

The *labor market tightness* θ_t is defined as the ratio of *competing* firms to unemployed workers, $\theta_t \equiv \phi_t(1 - G(x_t^c))$. The number of competing firms approaching a given unemployed worker in period t is a Poisson random variable with parameter θ_t .

If a firm with permanent productivity x hires a worker, the match produces output in period t equal to $A_t x$ where A_t is an exogenous shock that hits the economy at the start of period t . We refer to A_t as an *aggregate TFP shock*.

Matches are destroyed at the start of each period at an exogenous rate $\delta \in (0, 1]$.⁴ For tractability, we assume that once a worker and a firm agree to form a match, it can *only* terminate through exogenous match destruction.⁵ When a match is destroyed, the worker becomes unemployed and the firm's capital is destroyed.

Since the number of firms competing for each worker is a Poisson random variable, any given meeting between workers and firms can be either *bilateral* (i.e. exactly one competing firm) or *multilateral* (i.e. two or more competing firms).

In a multilateral meeting in period τ , firms compete to hire the worker. The highest productivity firm hires the worker and in each period $t \geq \tau$ until match destruction it produces output $A_t x^1$ where x^1 is the permanent productivity of the most productive firm. Since there are competing offers, the worker is paid $A_t x^2$ where x^2 is the permanent productivity of the second most productive firm competing for that worker.

In a bilateral meeting in period τ , where the firm has permanent productivity x , the firm hires the worker and in each period $t \geq \tau$ until match destruction it produces output $A_t x$. Since the worker has no other wage offers, he is simply paid his reservation wage b_τ at the time of hiring each period until match destruction.

If no firms approach an unemployed worker in period t , he receives the value of non-market activity $z \in [0, x_{\min}]$ and stays unemployed. For simplicity, we assume that a firm's capital is destroyed when it is unsuccessful in hiring.

The timing of events within each period t can be summarized as follows: (i) aggregate shocks A_t and C_t hit the economy and exogenous match destruction occurs; (ii) potential firms make an entry decision and pay the cost C_t ; (iii) search takes place and meetings between firms and workers occur; (iv) firms draw idiosyncratic permanent match-specific productivities and compete for workers; and (v) successful firms hire workers, produce output, pay wages, and earn profits.

²If firms draw productivities before deciding whether to search for a worker, the results are identical.

³We show in the Appendix that there exists a unique cut-off productivity x_t^c such that all matches with permanent productivity $x \geq x_t^c$ are acceptable to both workers and firms in period t .

⁴Match destruction applies to matches existing at the start of the previous period, not new hires.

⁵In the calibrated version of our model, we verify that the surplus of all matches is always positive.

1 *2.1. Unemployment, productivity, output, and factor shares*

2 Since the number of competing firms targeting each unemployed worker during period t is a
3 Poisson random variable with parameter θ_t , the meeting technology is urn-ball and therefore the
4 matching probability for unemployed workers and competing firms respectively is $m(\theta_t) = 1 - e^{-\theta_t}$
5 and $q(\theta_t) \equiv m(\theta_t)/\theta_t$.

6 The measure of unemployed workers U_t at the start of period t evolves according to the following
7 standard law of motion:

$$U_{t+1} = U_t(1 - m(\theta_t)) + \delta(L - U_t) \quad (1)$$

8 and the measure of employed workers N_t at the start of period t evolves according to

$$N_{t+1} = (1 - \delta)N_t + m(\theta_t)U_t. \quad (2)$$

9 The measure of *existing matches* that survive match destruction at the start of period $t + 1$ is
10 $(1 - \delta)N_t$ and the measure of *new hires* created in period t is $h_t \equiv m(\theta_t)U_t$. The unemployment
11 rate u_t at the start of period t is given by $u_t = U_t/L$.

12 *2.1.1. Endogenous match output*

13 In any given period, workers who are initially unemployed will either form a match with
14 permanent productivity x or they will remain unemployed and produce zero output that period.
15 We can derive the endogenous distribution of permanent match productivities x across workers
16 who are initially unemployed at the start of period t .

17 First, we define the (truncated) distribution of productivities among *competing* firms, $G_t^c(x) \equiv$
18 $\Pr(X < x | x \geq x_t^c)$. The minimum of this distribution is $x_{0t} \equiv \max\{x_{\min}, x_t^c\}$ since only firms with
19 productivity greater than x_t^c compete.

20 Suppose that n firms compete to hire a given worker in period t . The cdf of a worker's output
21 conditional on n firms competing is $H_t(x|n) = (G_t^c(x))^n$, the distribution of the maximum of n
22 draws from $G_t^c(x)$. To obtain $H_t(x; \theta_t)$, the endogenous distribution of productivities x across all
23 workers initially unemployed at time t , the conditional cdf $H_t(x|n)$ is weighted by the Poisson
24 probability that n firms compete:

$$H_t(x; \theta_t) = \sum_{n=0}^{\infty} \frac{\theta_t^n e^{-\theta_t}}{n!} (G_t^c(x))^n = e^{-\theta_t(1-G_t^c(x))} \quad (3)$$

25 The distribution $H_t(x; \theta_t) = e^{-\theta_t(1-G_t^c(x))}$ automatically builds in the possibility of unemploy-
26 ment, since the mass point at $x = 0$ with probability mass $1 - m(\theta_t) = e^{-\theta_t}$ represents initially
27 unemployed workers who remain unemployed (i.e. who are not matched at time t). We can also
28 define $H_t^e(x; \theta_t) \equiv (H_t(x; \theta_t) - e^{-\theta_t})/m(\theta_t)$, the distribution of productivities across workers who
29 are newly employed at time t .

30 We can define $p(\theta_\tau) \equiv E_{H_\tau^e}(x)$, the expected permanent productivity of a worker/firm match
31 created at time τ , given by

$$p(\theta_\tau) = \int_{x_{0\tau}}^{\infty} x dH_\tau^e(x; \theta_\tau). \quad (4)$$

32 Since $p(\theta_\tau) \equiv E_{H_\tau^e}(x)$ where $H_\tau^e(x; \theta_\tau)$ is defined above, expected match productivity de-
33 pends on labor market conditions *at the time τ of match creation*. This includes both the cut-off
34 productivity x_τ^c and the market tightness θ_τ , which affect match productivity through two differ-
35 ent channels. Greater market tightness means greater competition between firms to hire workers,
36 which allows workers to be more selective regarding firms and increases the average match produc-
37 tivity. A greater cut-off productivity x_τ^c means that only more productive firms enter, increasing
38 the average match productivity.

39 *2.1.2. Aggregate output*

40 With aggregate productivity shocks, the output of a match at time t is $A_t x$ where x is the
41 permanent component of match productivity and A_t is the aggregate TFP shock at time t . The

1 expected output at time t across all matches formed in period τ is given by $A_t p(\theta_\tau)$. We call this
 2 the *labor productivity* of cohort τ .

3 Aggregate output Y_t during period t is given by the following:

$$Y_t = A_t \sum_{a=0}^{\infty} v_{t,t-a} p(\theta_{t-a}) \quad (5)$$

4 where $v_{t,t-a} \equiv (1-\delta)^a h_{t-a}$, the *cohort measure* at time t of matches of age a that are still active.
 5 The expression for aggregate output Y_t weights the labor productivity $A_t p(\theta_{t-a})$ for matches
 6 created during period $t-a$ by the cohort measure $v_{t,t-a}$. The model has “vintage” features since
 7 matches of age a are permanently affected by the labor market conditions at the time of hiring.

8 2.1.3. Aggregate labor share

9 Letting $\tilde{w}_{t,\tau}$ be the average wage during period t for a worker hired during time τ , the average
 10 labor share $\tilde{s}_{L,t,\tau}$ during period t for matches created during time τ is

$$\tilde{s}_{L,t,\tau} = \frac{\tilde{w}_{t,\tau}}{A_t p(\theta_\tau)}. \quad (6)$$

11 The aggregate labor share $s_{L,t}$ during time t is therefore

$$s_{L,t} = \frac{\sum_{a=0}^{\infty} v_{t,t-a} \tilde{w}_{t,t-a}}{Y_t}. \quad (7)$$

12 We can also express the aggregate labor share as a weighted average of the cohort-specific
 13 labor shares $s_{L,t,\tau}$. Let $\tilde{v}_{t,\tau}$ be the *output share* at time t of cohort τ ,

$$\tilde{v}_{t,\tau} = \frac{v_{t,\tau} A_t p(\theta_\tau)}{Y_t}. \quad (8)$$

14 Clearly, we have $\sum_{a=0}^{\infty} \tilde{v}_{t,t-a} = 1$. Substituting (8) into (7) and using (8), we have

$$s_{L,t} = \sum_{a=0}^{\infty} \tilde{v}_{t,t-a} s_{L,t,t-a}. \quad (9)$$

15 That is, the aggregate labor share is a weighted average of the cohort-specific labor shares $s_{L,t,\tau}$,
 16 where each cohort τ is weighted by its output share $\tilde{v}_{t,\tau}$.

17 2.2. Asset value equations

18 Letting V_t^u be the value of unemployment at time t , the expected value $V_{t,\tau}^e$ of *employment* in
 19 period t in a match formed during period τ is

$$V_{t,\tau}^e = \tilde{w}_{t,\tau} + \beta((1-\delta)\mathbb{E}_t V_{t+1,\tau}^e + \delta\mathbb{E}_t V_{t+1}^u), \quad (10)$$

20 and the value of *unemployment* V_t^u is given by

$$V_t^u = m(\theta_t) V_{t,t}^e + (1-m(\theta_t))(z + \beta\mathbb{E}_t V_{t+1}^u). \quad (11)$$

21 Letting $J_{t,\tau}$ be the expected value in period t of a *filled vacancy* when the match was created
 22 in period τ , we have

$$J_{t,\tau} = A_t p(\theta_\tau) - \tilde{w}_{t,\tau} + \beta(1-\delta)\mathbb{E}_t J_{t+1,\tau}. \quad (12)$$

23 Since $q(\theta_\tau)$ is the probability a competing firm successfully hires a worker, the value V_τ of an
 24 *unfilled vacancy* (at the time of entry τ) is

$$V_\tau = -C_\tau + (1-G(x_\tau^c))q(\theta_\tau)J_{\tau,\tau}. \quad (13)$$

1 *2.3. Wage determination*

2 In this environment, both the permanent match productivity and wages depend crucially on
3 whether or not there is direct competition to hire the worker at the time of match creation, i.e.
4 on whether the meeting is *bilateral* or *multilateral*. Since the number of firms approaching a given
5 worker is a Poisson random variable with parameter θ , whether a worker faces a bilateral or a
6 multilateral meeting is random. However, the *proportion* of bilateral meetings, $\mu(\theta) \equiv \theta e^{-\theta}/m(\theta)$,
7 is endogenous. Since $\mu'(\theta) < 0$, greater firm competition implies a lower share of bilateral meetings.

8 Suppose a worker is hired in a *bilateral* meeting (i.e. exactly one competing firm) in period
9 τ . The worker is simply paid his reservation wage b_τ every period until match destruction. The
10 reservation wage b_τ is the lowest wage offer the worker is willing to accept: it satisfies the indiffer-
11 ence condition (14) that equates the expected payoff from accepting the wage offer and rejecting
12 it. That is,

$$V_{\tau,\tau}^1(b_\tau) = z + \beta \mathbb{E}_\tau V_{\tau+1}^u, \quad (14)$$

13 where the value $V_{t,\tau}^1(w)$ at time t of being employed at wage w in a bilateral match formed in
14 period τ is

$$V_{t,\tau}^1(w) = w + \beta((1 - \delta)\mathbb{E}_t V_{t+1,\tau}^1(w) + \delta \mathbb{E}_t V_{t+1}^u). \quad (15)$$

15 The expected value at time t of a *filled vacancy* with permanent productivity x that was created
16 in a bilateral meeting during period τ is

$$J_{t,\tau}^1(x) = A_t x - b_\tau + \beta(1 - \delta)\mathbb{E}_t J_{t+1,\tau}^1(x). \quad (16)$$

17 Suppose a worker is hired in a *multilateral* meeting (i.e. two or more competing firms) in
18 period τ . The firm with the highest productivity x hires the worker and pays a wage equal to
19 the second-highest productivity, indexed by the aggregate TFP shock A_t , each period until match
20 destruction.⁶ More precisely, the expected value of employment in period t in a match with
21 permanent productivity x for a worker hired in a multilateral meeting in period τ is

$$V_{t,\tau}^2(x) = w_{t,\tau}^2(A_t x) + \beta((1 - \delta)\mathbb{E}_t V_{t+1,\tau}^2(x) + \delta \mathbb{E}_t V_{t+1}^u), \quad (17)$$

22 where $w_{t,\tau}^2(A_t x)$ is the expected wage in period t . The expected value of employment in period t
23 in any match formed in a multilateral meeting in period τ is

$$V_{t,\tau}^2 = \tilde{w}_{t,\tau}^2 + \beta((1 - \delta)\mathbb{E}_t V_{t+1,\tau}^2 + \delta \mathbb{E}_t V_{t+1}^u), \quad (18)$$

24 where $\tilde{w}_{t,\tau}^2$ is the expected wage at time t for a worker hired in such a meeting. The expected value
25 at time t of the *filled vacancy* with permanent productivity x that was created in a multilateral
26 meeting during period τ is

$$J_{t,\tau}^2(x) = A_t x - w_{t,\tau}^2(A_t x) + \beta(1 - \delta)\mathbb{E}_t J_{t+1,\tau}^2(x). \quad (19)$$

27 In the Appendix, we verify that for all t there exists a unique *cut-off productivity* x_t^c such that
28 all matches formed in period t with $x \geq x_t^c$ are accepted by both workers and firms, and we show
29 that x_t^c satisfies $J_{t,t}^1(x) = 0$.

30 *2.3.1. Expected wages and labor share*

31 Firm profits consist of both *productivity rents* and *matching rents*, as discussed in Mangin
32 (2017). In the dynamic setting here, we derive in the Appendix the following expression for the
33 expected value of a filled vacancy:

$$J_{t,\tau} = \underbrace{\pi_{t,\tau}(\theta_\tau)}_{\text{productivity rents}} + \underbrace{\mu(\theta_\tau)(A_t x_{0\tau} - b_\tau)}_{\text{matching rents}} + \beta(1 - \delta)\mathbb{E}_t J_{t+1,\tau}, \quad (20)$$

⁶This wage determination mechanism is similar to Bertrand competition or second-price auctions. For a more detailed microfoundation and a discussion of the relevant literature see Mangin (2017).

1 where $\mu(\theta_\tau)$ is the share of bilateral meetings at time τ , and $\pi_{t,\tau}(\theta_\tau)$ is the expected value of
 2 productivity rents at time t for a match formed in period τ , defined by

$$\pi_{t,\tau}(\theta_\tau) \equiv \int_{A_t x_{0\tau}}^{\infty} \left(\frac{1 - G_{t,\tau}(y)}{y g_{t,\tau}(y)} \right) y dH_{t,\tau}^e(y; \theta_\tau), \quad (21)$$

3 where $G_{t,\tau}(y) \equiv G_\tau^c(y/A_t)$, and $H_{t,\tau}^e(y; \theta_\tau) \equiv H_\tau^e(y/A_t; \theta_\tau)$.

4 *Productivity rents* $\pi_{t,\tau}(\theta_\tau)$ arise because successful firms receive the difference between the
 5 highest and second highest productivity among the firms competing to hire the worker at time τ
 6 of hiring, multiplied by the current TFP shock A_t . The expected value of the productivity rents
 7 available to firms depends on properties of the underlying distribution G and is captured by (21).
 8 When there is no heterogeneity in firm productivities, i.e. when $G(x)$ is degenerate, $\pi_{t,\tau}(\theta_\tau) = 0$.

9 *Matching rents* reflect the flow match surplus for the *least* productive firm. Even if a competing
 10 firm has the lowest possible productivity $x_{0\tau}$, it can still earn profits if matched with a worker
 11 in a bilateral meeting. The value of matching rents in period t for matches created in period τ
 12 is $A_t x_{0\tau} - b_\tau$, which is reflected in the second term inside the brackets in (20). With aggregate
 13 productivity shocks, matching rents can in principle be either positive or negative in period t
 14 depending on the realization of the shock A_t . When $A_t x_{0\tau} - b_\tau = 0$, there are no matching rents.

15 Using (12) and (20), expected wages $\tilde{w}_{t,\tau}$ at time t of a match created at time τ is

$$\tilde{w}_{t,\tau} = \underbrace{A_t p(\theta_\tau)}_{\text{expected match output}} - \underbrace{\pi_{t,\tau}(\theta_\tau)}_{\text{productivity rents}} - \underbrace{\mu(\theta_\tau)(A_t x_{0\tau} - b_\tau)}_{\text{matching rents}}. \quad (22)$$

16 Next, using (6), the labor share $\tilde{s}_{L,t,\tau}$ at time t for matches created in period τ is

$$\tilde{s}_{L,t,\tau} = 1 - \underbrace{\frac{\pi_{t,\tau}(\theta_\tau)}{A_t p(\theta_\tau)}}_{\text{productivity rents}} - \underbrace{\frac{\mu(\theta_\tau)(A_t x_{0\tau} - b_\tau)}{A_t p(\theta_\tau)}}_{\text{matching rents}}, \quad (23)$$

17 where $p(\theta_\tau)$ is given by (4). In expression (23), productivity rents and matching rents are expressed
 18 as a *share* of match output. Using (7), the aggregate labor share is

$$s_{L,t} = 1 - \underbrace{\frac{\sum_{a=0}^{\infty} v_{t,t-a} \pi_{t,t-a}(\theta_{t-a})}{Y_t}}_{\text{productivity rents}} - \underbrace{\frac{\sum_{a=0}^{\infty} v_{t,t-a} \mu(\theta_{t-a})(A_t x_{0t-a} - b_{t-a})}{Y_t}}_{\text{matching rents}}. \quad (24)$$

19 Essentially, the aggregate capital share $s_{K,t}$ at time t is equal to the sum of both productivity
 20 rents and matching rents earned by firms in each cohort $\tau = t - a$, weighted by the relevant
 21 cohort measure $v_{t,t-a}$ and then expressed as a share of output. The aggregate labor share is
 22 $s_{L,t} = 1 - s_{K,t}$, which is given by (24).

23 2.3.2. Equilibrium definition

24 We can now present a formal definition of an equilibrium.

25 **Definition 1.** *Given exogenous stochastic processes for the two aggregate shocks A_t and C_t , an*
 26 *equilibrium is a sequence $\{\phi_t, x_t^c, b_t, U_t\}$ which satisfies*

- 27 • *the free entry condition, $V_{t,t} = 0$, i.e. $C_t = (1 - G(x_t^c))q(\theta_t)J_{t,t}$,*
- 28 • *the firm indifference condition, $J_{t,t}^1(x_t^c) = 0$,*
- 29 • *the worker indifference condition, $V_{t,t}^1(b_t) = z + \beta \mathbb{E}_t V_{t+1}^u$, and*
- 30 • *the law of motion (1) for unemployment U_t ,*

31 *where $\theta_t = \phi_t(1 - G(x_t^c))$, $J_{t,t}$ is given by (20), $J_{t,t}^1(x)$ is given by (16), $V_{t,t}^1(w)$ is given by (15),*
 32 *and V_t^u is given by (11).*

1 *2.4. Pareto distribution*

2 To obtain tractable expressions for aggregate output, wages, and factor income shares we focus
 3 on the Pareto distribution $G(x)$. Let $G(x) = 1 - x^{-1/\lambda}$ for $x \in [1, \infty)$ and $G(x) = 0$ otherwise.
 4 The parameter $\lambda \in (0, 1)$ is called the *shape parameter* and it governs the degree of productivity
 5 dispersion. For simplicity, we assume that λ is constant. Both the mean and the variance of $G(x)$
 6 are increasing in λ .

7 Given the reservation wage b_t , the (truncated) distribution of *competing* firms' permanent
 8 productivity levels, $G_t^c(x)$, remains Pareto, $G_t^c(x) = 1 - \left(\frac{x}{x_{0t}}\right)^{-1/\lambda}$ where $x_{0t} \equiv \max\{1, x_t^c\}$. For
 9 the Pareto distribution, we have

$$H_t(x; \theta_t) = \begin{cases} e^{-\theta\left(\frac{x}{x_{0t}}\right)^{-1/\lambda}} & \text{if } x \in [x_{0t}, \infty) \\ e^{-\theta_t} & \text{otherwise} \end{cases} \quad (25)$$

10 Except for the mass point at zero, the distribution $H_t(x; \theta_t)$ is similar to a Type II Extreme
 11 Value Distribution or Fréchet distribution. This is useful because it means that the distribution
 12 of realized match productivities is approximately Fréchet – a distribution that is often used to
 13 represent the firm productivity distribution.

14 We now present the specific expressions for wages and the labor share for the Pareto distribution
 15 using the general expressions found in Section 2.3.1. These expressions become very tractable
 16 because the Pareto distribution has the useful property that $\pi_{t,\tau}(\theta_\tau) = \lambda A_t p(\theta_\tau)$ for all t and
 17 τ . That is, a firm's expected productivity rents are a constant fraction λ of expected match
 18 output. In the absence of matching rents, this property would lead to constancy of the labor
 19 share. However, in the presence of matching rents, the labor share fluctuates over the business
 20 cycle.

21 The Pareto distribution allows us to decompose the labor share into two components: a con-
 22 stant part (due to productivity rents) and a variable part (matching rents). First, substituting
 23 $\pi_{t,\tau}(\theta_\tau) = \lambda A_t p(\theta_\tau)$ into (22) yields

$$\tilde{w}_{t,\tau} = \underbrace{(1 - \lambda) A_t p(\theta_\tau)}_{\text{expected output minus productivity rents}} - \underbrace{\mu(\theta_\tau)(A_t x_{0\tau} - b_\tau)}_{\text{matching rents}}. \quad (26)$$

24 Next, using (23), labor's share $\tilde{s}_{L,t,\tau}$ during period t for matches created in period τ is

$$\tilde{s}_{L,t,\tau} = 1 - \underbrace{\lambda}_{\text{productivity rents}} - \underbrace{\frac{\mu(\theta_\tau)(A_t x_{0\tau} - b_\tau)}{A_t p(\theta_\tau)}}_{\text{matching rents}}, \quad (27)$$

25 Substituting $\pi_{t,\tau}(\theta_\tau) = \lambda A_t p(\theta_\tau)$ into (24), the aggregate labor share $s_{L,t}$ at time t is

$$s_{L,t} = 1 - \underbrace{\lambda}_{\text{productivity rents}} - \underbrace{\frac{\sum_{a=0}^{\infty} v_{t,t-a} \mu(\theta_{t-a})(A_t x_{0t-a} - b_{t-a})}{Y_t}}_{\text{matching rents}}. \quad (28)$$

26 Since average productivity rents are a constant fraction of output, the aggregate labor share simply
 27 equals a constant term $1 - \lambda$ minus the sum of matching rents – weighted by the relevant cohort
 28 measure – as a share of output. This decomposition makes it clear that it is matching rents that
 29 drive the fluctuations in the labor share.

30 Labor's share $s_{L,t}$ is constant (i.e. depends only on the parameter λ) in a given period t if and
 31 only if $x_\tau^c \geq x_{\min}$ and therefore $x_{0\tau} = x_\tau^c$, and, in addition $x_\tau^c = b_\tau/A_t$ for *all cohorts* τ . In this
 32 case, the value of matching rents, $A_t x_{0\tau} - b_\tau$, is always zero and labor's share is constant because
 33 the average productivity rents are linear in output. Clearly, this extreme case is not possible in
 34 the presence of aggregate TFP shocks. Even for a given cohort τ , labor's share $s_{L,t,\tau}$ can only be
 35 constant in rare periods where matching rents, $A_t x_{0\tau} - b_\tau$, are zero. Therefore, with TFP shocks,
 36 both the aggregate labor share and cohort-specific labor shares always fluctuate over time.

3. Steady state

Consider a stationary environment where $A_t = A$, $z_t = z$ and $C_t = C$ for all t . Let $U_{t+1} = U_t = U$ and $\theta_{t+1} = \theta_t = \theta$ for all t . Let $G(x) = 1 - x^{-1/\lambda}$ for $x \in [1, \infty)$ and $G(x) = 0$ otherwise, i.e. $x_{\min} = 1$. Given the reservation wage b , the distribution of *competing* firms' productivity levels is $G^c(x) = 1 - \left(\frac{x}{x_0}\right)^{-1/\lambda}$ where $x_0 = \max\{1, x^c\}$. In the steady state, we have $J_1(x^c) = 0$ and only if $x^c = b/A$.⁷

Steady state average match productivity is

$$p(\theta) = \frac{x_0 \gamma(1 - \lambda, \theta) \theta^\lambda}{1 - e^{-\theta}}, \quad (29)$$

where $\gamma(s, x)$ is the Lower Incomplete Gamma Function defined by $\gamma(s, x) \equiv \int_0^x t^{s-1} e^{-t} dt$.⁸ Average match productivity $p(\theta)$ is increasing in θ (taking b as given). Since the Pareto distribution is *well-behaved*, this follows from Lemma 2 in Mangin (2017).

Steady state output is given by

$$Y = \frac{Ax_0 \gamma(1 - \lambda, \theta) \theta^\lambda U}{\delta}. \quad (30)$$

We can interpret expression (30) as an aggregate production function. To understand it better, consider the special case where $\delta = 1$ and $b \leq A$. Since each firm has one unit of capital, $K = V$ in this case and $U = L$, so we have $Y = A\gamma(1 - \lambda, \theta) K^\lambda L^{1-\lambda}$. In the limit where $\theta \rightarrow \infty$ and unemployment disappears, this function is asymptotically Cobb-Douglas, $Y = A\Gamma(1 - \lambda) K^\lambda L^{1-\lambda}$, where $\Gamma(s)$ is the Gamma function.⁹

3.1. Steady state factor shares

Steady state labor share s_L is

$$s_L = 1 - \lambda - \frac{\mu(\theta)(Ax_0 - b)}{Ap(\theta)}. \quad (31)$$

Steady state factor shares are “constant” (i.e. depend only on λ) in two special cases. First, there is an asymptotic result: factor shares are constant in the limit as $\theta \rightarrow \infty$ and unemployment disappears. In this case, all firms face direct competition and the *probability* of receiving matching rents disappears because the share of bilateral meetings goes to zero, i.e. $\mu(\theta) \rightarrow 0$. Second, factor shares are constant when $x_0 = b/A$. Since $x_0 = \max\{1, x^c\}$, this means that factor shares are constant when $x^c = b/A \geq x_{\min} = 1$, i.e. when $b \geq A$. In this case, the *value* of matching rents disappears since $Ax_0 - b = 0$. While the first case cannot arise as an equilibrium outcome, the second case can in principle arise as an equilibrium outcome.¹⁰

The constancy of steady state factor shares in both special cases arises from the fact that $\pi(\theta) = \lambda Ap(\theta)$. That is, productivity rents – equal to the expected difference between the highest and the second highest productivity among competing firms – are a constant fraction of the expected match output.¹¹ This result applies *only* in the steady state. As discussed in Section 2.4, the existence of aggregate productivity shocks A_t implies that factor shares always fluctuate regardless of the reservation wage.

⁷Using (16), $J^1(x) = (Ax - b)/(1 - \beta(1 - \delta))$ and hence $J^1(x) = 0$ if and only if $x = b/A$.

⁸See the Appendix for some useful properties of the function $\gamma(s, x)$.

⁹This limiting aggregation result is related to Jones (2005), who derives a Cobb-Douglas aggregate production function using a Pareto distribution of ideas. Lagos (2006) also derives a Cobb-Douglas aggregate production function using the Pareto distribution in a Mortensen-Pissarides style search-theoretic model with random matching and Nash bargaining. See Mangin (2017) for details.

¹⁰In Section 3.2, we will see that this is because as $\theta \rightarrow \infty$ and hence $\phi \rightarrow \infty$, we have $b_r(\phi) > \bar{b}$ but $\phi_r(b) = 0$ when $b > \bar{b}$.

¹¹Importantly, this property is unique to the Pareto distribution. If $G(x)$ is not Pareto, this is not the case and therefore factor shares are not constant when $b \geq A$. For example, in the Appendix we derive an expression for the steady state labor share using the Generalized Pareto Distribution.

1 **Lemma 1.** *Steady state labor share is increasing in the degree of firm competition θ .*

2 In general, steady state labor share is increasing in the degree of firm competition, as measured
 3 by the market tightness θ (taking b as given). Intuitively, as the number of competing firms per
 4 unemployed worker θ rises, greater competition leads to an increase in labor's share. In particular,
 5 the share of bilateral meetings is lower ($\mu'(\theta) < 0$) so the probability of direct competition to hire
 6 a worker is greater. At the same time, average match productivity $p(\theta)$ increases and therefore
 7 $\mu(\theta)/p(\theta)$ is decreasing in θ .

8 It is clear that labor's share is increasing in the reservation wage b (taking θ as given) since in
 9 bilateral meetings workers are paid more when their reservation wage is higher, thereby decreasing
 10 the value of the matching rents, $Ax_0 - b$, available to firms.

11 3.2. Steady state equilibrium

12 A steady state equilibrium can be characterized by a triple (ϕ, b, U) that satisfies Definition 1
 13 where $x^c = b/A$ and $\theta = \phi(1 - G(x^c))$.

14 For the Pareto distribution, the zero profit condition for firms is

$$C = \frac{x_0^{-1/\lambda} q(\theta) (\pi(\theta) + \mu(\theta)(Ax_0 - b))}{1 - \beta(1 - \delta)}. \quad (32)$$

15 In the Appendix, we show that Assumption 2 implies that there is a critical value $\bar{b}(\lambda, \beta, \delta, C) >$
 16 z such that for any $b < \bar{b}$, there exists a unique level of firm entry $\phi > 0$ which satisfies (32). If
 17 $b \geq \bar{b}$, there is no firm entry and hence $\phi = \theta = 0$. So for any given $b \geq 0$, there is a unique level
 18 of firm entry ϕ and we have a function $\phi_r : \mathbb{R}^+ \rightarrow \mathbb{R}^+$. When $b < \bar{b}$, this function is differentiable
 19 and we show that $\phi'_r(b) < 0$. Since $\phi = 0$ when $b \geq \bar{b}$, ϕ_r is weakly decreasing in b .

20 **Assumption 2.** *The cost of purchasing capital is not too high:*

$$C < \frac{1}{1 - \beta(1 - \delta)} \left(\frac{A}{1 - \lambda} - z \right). \quad (33)$$

21 Let V^u be the steady state value of unemployment and let $V^1(w)$ be the steady state value
 22 of employment at wage w in a bilateral meeting. In a bilateral meeting, unemployed workers will
 23 accept a job offer at wage w if $V^1(w) \geq z + \beta V^u$ and reject it otherwise. In the Appendix, we verify
 24 that for any given $\phi \geq 0$, there exists a unique reservation wage b that satisfies the indifference
 25 condition:

$$V^1(b) = z + \beta V^u. \quad (34)$$

26 So we have a function $b_r : \mathbb{R}^+ \rightarrow \mathbb{R}^+$, and we verify that $b'_r(\phi) > 0$ for all ϕ .

27 Substituting $V^1(b)$ and V^u from (15) and (11) into (34) yields the following expression for the
 28 steady state reservation wage,

$$b = \frac{z(1 - \beta(1 - \delta)) + \beta(1 - \delta)w(\theta, b)}{1 - \beta(1 - \delta)e^{-\theta}}, \quad (35)$$

29 where $w(\theta, b) \equiv \tilde{w}m(\theta)$. The reservation wage b is a weighted average of the flow value z of
 30 non-market activity and the expected wage for *all* workers (including the unemployed). It is clear
 31 from (35) that when $\phi = \theta = 0$, we have $b_r(0) = z$.

32 **Proposition 1.** *There is a unique steady state equilibrium (ϕ^*, b^*) where $z \leq b^* < \bar{b}$.*

33 We know that for any given $\phi \geq 0$, (34) has a unique solution $b_r(\phi)$. At the same time, we
 34 know that for any given $b < \bar{b}$, (32) has a unique solution $\phi_r(b)$, and if $b \geq \bar{b}$ we have $\phi_r(b) = 0$.
 35 Since the function b_r is increasing in ϕ , and ϕ_r is decreasing in b , there exists a unique steady
 36 state equilibrium (ϕ^*, b^*) that satisfies both (34) and (32). Since $b_r(0) = z$ and $b'_r(\phi) > 0$, we have
 37 $b_r(\phi) \geq z$ for all ϕ .

1 *3.3. Comparative statics*

2 Proposition 2 contains some comparative statics results regarding the effects of the key pa-
 3 rameters – the value of non-market activity z , the cost of purchasing capital C , and the shape
 4 parameter λ from the underlying productivity distribution – on the equilibrium (ϕ^*, b^*) , as well
 5 as on the steady state equilibrium unemployment rate, $u^* \equiv u(\theta^*)$, output per capita $y^* \equiv Y/L$,
 6 and labor productivity $p^* \equiv Y/N$. We focus on the case where $b^*/A < x_{\min} = 1$ and therefore
 7 $x_0 = 1$ and $\theta^* = \phi^*$ because we are primarily interested in the behavior of factor shares.

8 **Proposition 2.** *If $b^* < x_{\min}$, then: (i) workers' reservation wage b^* is increasing in λ , z , and A ,
 9 and decreasing in C ; (ii) the labor market tightness θ^* is decreasing in z and C , and increasing
 10 in λ and A ; (iii) the unemployment rate u^* is increasing in both z and C , and decreasing in λ
 11 and A ; (iv) output per capita is decreasing in z and C , and increasing in λ and A ; and (v) labor
 12 productivity p^* is decreasing in z and C , and increasing in λ and A .*

13 Intuitively, an increase in the value of non-market activity z leads to a direct increase in the
 14 reservation wage and a resulting decrease in the market tightness. An increase in the cost of
 15 capital C leads to a decrease in the market tightness as less firms enter, resulting in a decrease in
 16 the reservation wage. In both cases, less firm competition results in higher unemployment as well
 17 as lower labor productivity and lower output per capita. By contrast, an increase in the level of
 18 TFP A leads to an increase in the reservation wage and an increase in the market tightness. In this
 19 case, greater firm competition results in lower unemployment as well as higher labor productivity
 20 and higher output per capita. The effects of a change in the shape parameter λ are subtle.

21 Propositions 3, 4, and 5 provide some comparative statics for the labor share.

22 **Proposition 3.** *Steady state labor share s_L^* is increasing in the flow value of non-market activity
 23 z .*

24 An increase in the value of non-market activity z leads to an increase in the reservation wage,
 25 which has a positive direct effect on the labor share. There is also a negative indirect effect on
 26 the labor share through the labor market tightness θ^* , since a higher z decreases the level of firm
 27 entry, which has a negative impact on the labor share. Overall, the positive effect dominates and
 28 the labor share is increasing in z .

29 **Proposition 4.** *Steady state labor share s_L^* is decreasing in the capital cost C .*

30 When the cost of capital C increases, there is an indirect effect on the labor share through the
 31 market tightness θ^* . A higher cost C means that entry is less attractive for firms, decreasing θ^* .
 32 This has a negative effect on the labor share since there is less competition for workers. Overall,
 33 equilibrium labor share is decreasing in the cost C .

34 *3.3.1. Effect on labor share of an increase in TFP*

35 An increase in the level of TFP A influences the steady state labor share through two distinct
 36 channels. The first is through θ^* . Since θ^* is increasing in A by Proposition 2, and $\mu(\theta)/p(\theta)$ is
 37 decreasing in θ by Lemma 1, there is a positive effect on labor share through θ^* . The second is
 38 through the term b^*/Ax_0 in (31). If this term increases when A does, there is a positive impact
 39 on labor share. If this term decreases when A does, there is a negative impact on labor share.
 40 Overall, the effect of A on labor share depends on the degree of sensitivity of the reservation wage
 41 b^* to changes in A .

42 Before presenting a result regarding the effect of a change in the level of TFP A on the
 43 equilibrium labor share, we define some terms. Let $\varepsilon_{b^*}(A)$ be the elasticity of b^* with respect
 44 to TFP A , let $\varepsilon_{\theta^*}(A)$ be the elasticity of θ^* with respect to A , and let $\eta_\varepsilon(\theta)$ be the elasticity of
 45 $\mu(\theta)/p(\theta)$ with respect to labor market tightness θ .¹²

¹²In the Appendix, we provide analytic expressions for $\varepsilon_{b^*}(A)$, $\varepsilon_{\theta^*}(A)$, and $\eta_\varepsilon(\theta^*)$.

1 **Proposition 5.** *The equilibrium labor share s_L^* is increasing in the level of TFP A if and only if*
 2 *the reservation wage is sufficiently responsive to A :*

$$\varepsilon_{b^*}(A) > \frac{A}{b^*} \left(\frac{Ax_0 - b^*}{Ax_0} \right) \varepsilon_{\theta^*}(A) \eta_\varepsilon(\theta^*) + 1 \quad (36)$$

3 Since $\mu(\theta)/p(\theta)$ is decreasing in θ , we have $\eta_\varepsilon(\theta^*) < 0$ while $\varepsilon_{\theta^*}(A) > 0$, so a sufficient condition
 4 for labor share s_L^* to be increasing in A is that $\varepsilon_{b^*}(A) > 1$. Using the calibration in Section 4, we
 5 find that condition (36) holds at the steady state equilibrium. In fact, we find that $\varepsilon_{b^*}(A) = 1.41$,
 6 so the sufficient condition easily holds. Therefore, the steady state equilibrium labor share is
 7 increasing in TFP.

8 Later, when we calibrate the model and describe its dynamics in Section 4, we will see that this
 9 is similar to the impact of a positive TFP shock on *new hires*, but the effect on existing matches
 10 is different. As we will see, the different effects of a TFP shock on new vs existing matches gives
 11 rise to rich dynamics for the labor share.

12 3.3.2. Effect on labor share of a change in distribution G

13 We now consider the effect on the steady state equilibrium labor share of a change in the
 14 underlying productivity distribution G . For the Pareto distribution, we can change the distribution
 15 G by varying the shape parameter λ , which is a simple proxy for the degree of firm heterogeneity
 16 or productivity dispersion.¹³

17 When $b \geq A$, we have $s_L = 1 - \lambda$ and labor's share is clearly decreasing in λ . However, when
 18 $b < A$ the net effect is ambiguous. We can decompose the net effect into two opposing effects. For
 19 simplicity, we express it in terms of capital share, $s_K^* = 1 - s_L^*$.

$$\frac{ds_K^*}{d\lambda} = \underbrace{\frac{d}{d\lambda}(\lambda)}_{> 0 \text{ productivity rents}} + (Ax_0 - b^*) \underbrace{\left(\frac{d}{d\lambda} \frac{\mu(\theta^*)}{Ap(\theta^*)} \frac{\partial b^*}{\partial \lambda} \frac{\mu(\theta^*)}{Ap(\theta^*)} \right)}_{< 0 \text{ matching rents}}. \quad (37)$$

20 The productivity rents earned by firms, as a share of output, are clearly increasing in λ . For
 21 the Pareto distribution, productivity rents as a share of output are simply *equal* to the tail index
 22 parameter λ . This is intuitive: greater firm heterogeneity implies a greater expected value of the
 23 *difference* between the highest and second highest productivity, as a share of output.

24 The overall effect of λ on matching rents, as a share of output, is negative. First, $\partial b^*/\partial \lambda > 0$
 25 by Proposition 2, so the impact on matching rents through b^* is negative. Intuitively, workers
 26 increase their reservation wage when λ increases, which decreases the value of matching rents,
 27 $Ax_0 - b^*$, available to firms. Second, we have

$$\frac{d}{d\lambda} \frac{\mu(\theta^*)}{Ap(\theta^*)} = \underbrace{\frac{\partial \theta^*}{\partial \lambda} \frac{\partial}{\partial \theta} \frac{\mu(\theta^*)}{Ap(\theta^*)}}_{< 0 \text{ indirect effect}} + \underbrace{\frac{\partial}{\partial \lambda} \frac{\mu(\theta^*)}{Ap(\theta^*)}}_{< 0 \text{ direct effect}}. \quad (38)$$

28 Since equilibrium market tightness θ^* is increasing in λ by Proposition 2, and $\mu(\theta)/p(\theta)$ is de-
 29 creasing in θ according to Lemma 1, there is a negative indirect effect on matching rents of an
 30 increase in λ through θ . The direct effect of an increase in λ on matching rents is also negative
 31 since the denominator is increasing in λ .

32 Overall, the net effect on labor's share of an increase in λ depends on the relative size of
 33 the opposing effects on productivity rents and matching rents. Proposition 4 of Mangin (2017)
 34 provides a sufficient condition for equilibrium labor share to be increasing in λ in a static setting:
 35 the value of matching rents cannot be too high. If this condition is satisfied, the positive effect on

¹³Strictly speaking, an increase in λ is not a mean-preserving spread in G since it increases both the mean and variance. We have verified numerically, however, that the comparative static results are the same if we normalize $x_{\min} = 1 - \lambda$ so that an increase in λ is a true mean-preserving spread.

1 productivity rents of an increase in the parameter λ dominates the negative effect on matching
2 rents. However, this condition only applies in a static environment where $b^* = z$. In this case,
3 $\partial b^*/\partial \lambda$ equals zero and the second component of matching rents in (37) disappears.

4 In a dynamic environment where the reservation wage b^* is endogenous, the effect of λ on labor's
5 share is more complicated because the value of matching rents, $Ax_0 - b^*$, is also endogenous. We
6 cannot obtain simple analytical results; however, using the calibration described in Section 4,
7 we find that expression (37) is positive and therefore the steady state equilibrium labor share is
8 decreasing in the parameter λ .

9 4. Dynamics over time and in response to shocks

10 In this section, we analyze the model quantitatively with a particular focus on labor share
11 dynamics. Section 4.1 describes the calibration and the model's performance along several dimen-
12 sions related to the labor market. The remaining sections describe the model's dynamics, zooming
13 in on the behavior of the labor share. Section 4.2 describes the model's dynamics using impulse
14 response functions (IRFs) to the two aggregate shocks. Section 4.3 considers the business cycle
15 fluctuations in the labor share and, in particular, its dynamics following TFP shocks. Finally,
16 Section 4.4 examines to what extent the model can account for the observed time path of the U.S.
17 labor share. To do this, we use our calibrated model to *estimate* the underlying shocks.

18 4.1. Calibration and model performance

19 All model parameters and their respective targets are in Table 1. The model period is assumed
20 to be one month. We set the discount factor to 0.996, resulting in an annual interest rate of about
21 4 percent. The minimum value of the firm productivity distribution is normalized to $x_{\min} = 1$.
22 The shape parameter λ is crucial for determining the endogenous productivity distribution and
23 we therefore set λ such that the degree of productivity dispersion among *active* firms displays a
24 90 – 10 percentile range of 0.84, the midpoint between the empirical estimates in Syverson (2004).

25 In order to pin down the separation rate δ , the value of non-market activity z , and the average
26 cost of capital \bar{C} (which is assumed to be paid in units of output), we target a steady state
27 unemployment rate of 5.8 percent, a labor share of 68 percent, and a job finding probability of
28 45%. We use data from Rios-Rull and Santaella-Llopis (2010) who construct a measure of the
29 labor share for the period 1954Q1 to 2004Q4. The values for the unemployment rate, taken from
30 the Bureau of Labor Statistics, and the job finding probability, computed by Shimer (2005), are
31 averages over this period. Note that this calibration procedure does not take a stand on the value
32 of the replacement rate, for which there is little consensus in the literature. The replacement rate
33 implied by our calibration is about 35% of average wages ($z = 0.32$), which is similar to values in
34 e.g. Shimer (2005), Elsby and Michaels (2013), and Jung and Kuester (2015).

35 We consider two aggregate shocks: a TFP shock A_t and a shock to C_t . Since each firm owns
36 a unit of capital, the mass of existing firms also represents the capital stock and the number
37 of new firms represents capital investment. Therefore, we assume that capital costs are paid
38 in units of output and we interpret shocks to C_t as investment-specific technology shocks along
39 the lines of e.g. Fisher (2006) and Lopez-Salido and Michelacci (2007). Both aggregate shocks
40 are assumed to follow an AR(1) process.¹⁴ The persistence and standard deviations of the two
41 aggregate shocks are set such that the model replicates the persistence and volatility of output
42 and labor productivity in the data.¹⁵ This calibration results in the investment-specific technology
43 shocks being about 4.5 times as large as TFP shocks, in line with empirical estimates for the U.S.
44 economy (see e.g. Justiniano and Primiceri, 2008).

45 Table 2 documents the business cycle statistics resulting from the above calibration. It shows
46 that the model does relatively well in capturing both the volatility of unemployment and the
47 job finding probability relative to labor productivity, while the variation of vacancies is about 40
48 percent of that observed in the data. The co-movement of labor market variables in the model is
49 consistent with the data, including a strong Beveridge curve relationship.

¹⁴While the mean of the aggregate TFP shock is normalized to one, investment-specific technology shocks have a mean equal to the steady state capital cost \bar{C} .

¹⁵Labor productivity is measured as real output per worker in the non-farm business sector for the period of 1954Q1 to 2004Q4.

1 4.2. Aggregate labor market dynamics

2 Figure 1 shows the IRFs of several model variables to expansionary one-standard-deviation
3 shocks to both TFP and investment-specific technology. Following a positive TFP shock (an
4 increase in A_t), all employment relationships become more productive, raising aggregate output
5 and increasing the incentives to post vacancies (top and middle right panels). This is reflected in
6 an increase in the probability of finding a job and a decrease in the probability of filling a vacancy
7 (bottom panels) and it ultimately results in a fall in unemployment (middle left panel).

8 The same pattern is true also for an expansionary investment-specific technology shock (a
9 decrease in C_t). However, this shock “only” makes new employment relationships cheaper to start
10 up; it does not increase the productivity of existing matches. Therefore, the output response to
11 this shock is muted (top right panel).

12 The above discussion shows that the aggregate labor market dynamics of the model are similar
13 to those of a standard Diamond-Mortensen-Pissarides model. However, our model incorporates
14 novel features – the competition and cohort effects – which deliver richer dynamics. In particular,
15 average labor productivity is not simply equal to the aggregate TFP shock A_t , but it is influenced
16 by the composition of the employment pool with respect to firm-specific productivity levels and
17 labor market conditions at the time of hiring. Figure 2 shows the qualitatively different impact
18 the two aggregate shocks have on the dynamics of labor productivity.

19 While labor productivity increases and falls back to its steady state in a standard fashion
20 following an aggregate TFP shock, it initially *falls* in response to an expansionary investment-
21 specific technology shock. The reason for this initial drop is that cheaper capital promotes hiring
22 and increases the average productivity of *new* hires due to greater competition. However, the shock
23 leaves unchanged the productivity of existing matches, which dominate the economy initially. The
24 increase in employment combined with only a moderate rise in average match productivity results
25 in aggregate labor productivity falling in the first several quarters. Only after a sufficiently large
26 mass of new hires accumulates does aggregate labor productivity also rise.

27 Importantly, this initially negative response of labor productivity to investment-specific shocks
28 has been documented in existing studies (see e.g. Fisher, 2006; Canova et al., 2013). The mech-
29 anism behind these patterns rests on the presence of worker and firm cohort-effects driven by
30 aggregate conditions at the time of hiring. Such cohort effects have also been recently docu-
31 mented in the data. For example, Sedláček and Sterk (2017) document that startups born during
32 aggregate downturns remain smaller than their counterparts born in booms even after several
33 years of existence. In addition, Oreopoulos et al. (2012) show that college graduates experience a
34 9 percent earnings loss when they graduate in a recession, an effect largely due to unemployment
35 “shocks” experienced that year. We find that the above wage penalty is about 7 percent in our
36 model.¹⁶

37 4.3. Business cycle dynamics of the labor share

38 The labor share is counter-cyclical in the data. Moreover, Rios-Rull and Santaaulalia-Llopis
39 (2010) document that upon a positive TFP shock, the labor share initially drops but then *over-*
40 *shoots* after about five quarters before eventually settling back to its steady state.¹⁷ It is this
41 overshooting behavior that has proven difficult to understand through the lens of standard real
42 business cycle and search and matching models.

43 In the calibrated version of our model, as in the data, the labor share is *counter-cyclical*. The
44 correlation coefficient of the (logged and HP-filtered) labor share with aggregate output is -0.39 ,
45 while it is -0.22 in the data. As in standard search and matching models, this happens because
46 wages are somewhat rigid. In our model, this rigidity is not ad-hoc but micro-founded and stems
47 from the fact that wages partly reflect labor market conditions via competition at the time of
48 hiring. In addition to being counter-cyclical, the labor share in the model also lags output, as in

¹⁶The wage penalty described in Oreopoulos et al. (2012) is based on defining a recession as a period of time during which unemployment increases by 5 percentage points. Following this methodology, we compute the model-predicted wage penalty by simulating our model and averaging the wage differential of newly hired workers (compared to the aggregate) in periods when the unemployment rate rises by 5 percentage points.

¹⁷All empirical properties of the labor share are taken from Rios-Rull and Santaaulalia-Llopis (2010).

1 the data. The correlation coefficient peaks for an output lead of four quarters with a value of 0.4
 2 both in the model and the data.

3 Importantly, the model can account not only for the counter-cyclicality of the labor share but
 4 also its overshooting behavior. Figure 3 shows that the labor share drops upon impact of a positive
 5 TFP shock, but then rises quickly above its steady state level after about six quarters, i.e. the
 6 labor share *overshoots*. The timing of the overshooting behavior is almost identical to that in the
 7 data. Moreover, the magnitude of the response is also realistic, with the labor share dropping to
 8 about -0.2 percent on impact and peaking at about 0.2 percent in the data.¹⁸

9 4.3.1. Labor share for new hires and existing matches

10 The reason behind the labor share dynamics lies in the changing composition of the employment
 11 pool with respect to aggregate conditions at the time of hiring. To understand the labor share
 12 dynamics, we consider separately the roles of new hires and existing matches, i.e. those hired after
 13 and before the shock hit the economy.

14 Suppose that a positive TFP shock hits the economy in period T . Let $\alpha_{t,T} \equiv \sum_{a=0}^{t-T} \tilde{v}_{t,t-a}$,
 15 the *total* output share of new hires (i.e. those hired post-shock). For new hires, we can define
 16 $\hat{v}_{t,\tau} \equiv \tilde{v}_{t,\tau}/\alpha_{t,T}$, the output share of cohort τ among *new* hires. For existing matches (i.e. those
 17 hired pre-shock), we can define $\bar{v}_{t,\tau} \equiv \tilde{v}_{t,\tau}/(1 - \alpha_{t,T})$, the output share of cohort τ among *existing*
 18 matches. Rewriting (9), we have

$$s_{L,t} = \alpha_{t,T} \underbrace{\sum_{a=0}^{t-T} \hat{v}_{t,t-a} s_{L,t,t-a}}_{\text{labor share of new hires}} + (1 - \alpha_{t,T}) \underbrace{\sum_{a=t-T+1}^{\infty} \bar{v}_{t,t-a} s_{L,t,t-a}}_{\text{labor share of existing workers}} . \quad (39)$$

19 That is, the aggregate labor share is a weighted average of the labor shares for new hires and
 20 existing matches, which are themselves weighted averages of the cohort-specific labor shares $s_{L,t,\tau}$.
 21 The output share of new hires $\alpha_{t,T}$ measures the relative strength of the effects of a shock on new
 22 hires versus existing matches.

23 Since $x_{0\tau} = 1$ in our calibration, we can express the labor share for cohort τ as:

$$s_{L,t,\tau} = 1 - \lambda - \left(1 - \frac{b_\tau}{A_t}\right) \frac{\mu(\theta_\tau)}{p(\theta_\tau)}. \quad (40)$$

24 For any given cohort of $\tau < T$ of *existing* matches (pre-shock), all terms except for A_t are
 25 determined by aggregate conditions at the time of hiring τ , i.e. prior to the realization of the
 26 shock. Therefore, the impact of the shock is determined solely by the *direct* effect of A_t through
 27 the term b_τ/A_t and the labor share of existing workers falls.

28 For any given cohort $\tau \geq T$ of *new* hires (post-shock), the effect of the TFP shock is complex.
 29 For example, consider the effect on the labor share of matches created shortly after the shock hits.
 30 When A increases, there is a negative direct effect on labor share, as for existing matches. However,
 31 there are also *indirect* effects through the reservation wage b_τ and the market tightness θ_τ , both
 32 of which increase. Intuitively, the competition effect through θ_τ on labor share is positive, since
 33 $\mu(\theta)/p(\theta)$ is decreasing in θ . However, the net effect on labor share is ambiguous. The comparative
 34 static result in Proposition 5 suggests that the labor share will be increasing in a positive TFP
 35 shock provided that the elasticity of the reservation wage with respect to A is greater than one.
 36 Since this is true under our calibration ($\varepsilon_{b^*}(A) = 1.41$), we expect to see the labor share for new
 37 hires increase in response to a positive TFP shock.

38 Figure 4 decomposes the IRF for the aggregate labor share (top left panel) into its three
 39 determinants: the labor share for new hires (bottom left panel); the labor share for existing hires
 40 (bottom right panel); and the output share of new hires (top right panel). The two opposing
 41 effects on new and existing hires are key for understanding the aggregate labor share dynamics,
 42 and the output share of new hires indexes the relative strength of these two effects.

¹⁸The unconditional volatility of the labor share relative to that of labor productivity is somewhat larger in the model than in the data (0.79 in the model compared to 0.52 in the data).

1 In response to a positive TFP shock, the aggregate labor share initially drops (top left panel).
 2 This is because existing matches, which experience an immediate fall in the labor share (bottom
 3 right panel), initially dominate the economy: the output share of new hires $\alpha_{t,T}$ is low (top
 4 right panel). In contrast to existing matches, workers hired after the TFP shock are hired in an
 5 environment of stronger competition and they therefore experience an increase in their labor share
 6 (bottom left panel). As the share of workers hired under stronger competition $\alpha_{t,T}$ increases (top
 7 right panel), the aggregate labor share rises and eventually overshoots its steady state level. The
 8 hump-shaped response stems from the fact that wages of new hires not only rise relatively more
 9 than output, but they are also somewhat more persistent.

10 4.3.2. Wage elasticities

11 The above mechanism rests on the model prediction that wages of newly hired workers are
 12 more flexible than those in existing employment relationships. This prediction is consistent with
 13 existing studies (see e.g. Bils, 1985; Haefke et al., 2013).¹⁹ To investigate whether this mechanism
 14 is also *quantitatively* reasonable, we follow the methodology of the above studies and compute the
 15 wage elasticity with respect to aggregate labor productivity in the model.²⁰

16 The model predicts that while the wage elasticity for all workers is about one half, $\eta_{all} = 0.5$,
 17 that of new hires is around 1.5 times higher, $\eta_{new} = 0.7$. These values are consistent with the
 18 range of estimates found in existing studies.²¹ Therefore, the micro-founded distinction between
 19 new and existing matches – which is key to the model’s dynamics – is not only qualitatively but
 20 also quantitatively plausible.

21 4.4. Accounting for the time path of the U.S. labor share

22 We now investigate to what extent the model is able to account for the *time path* of the U.S.
 23 labor share during the period 1954Q1-2004Q4. In addition, we analyze how the novel features of
 24 our model – the competition and cohort effects – contribute to the model’s ability to account for
 25 the dynamics of the labor share.

26 We start by using our calibrated model to back out the time path of the two aggregate driving
 27 forces: the aggregate TFP (A_t) shock and the investment specific shock (C_t). To do so, we use
 28 data on real GDP and the unemployment rate. While both shocks affect the dynamics of the
 29 entire model, data on real GDP is primarily informative about the aggregate TFP shock – which
 30 directly impacts the output of all matches – and the time path of unemployment is closely related
 31 to investment-specific shocks.

32 The time series for both real GDP and the unemployment rate are logged and HP-filtered.
 33 Within the linearized model, the time path of the aggregate shocks are then estimated using the
 34 Kalman filter. By construction, the estimated model exactly matches the time path of aggregate
 35 output, given by equation (5), and the unemployment rate, given by equation (1). Importantly,
 36 the estimated shocks together with the model structure imply time paths for all other model
 37 variables, including the aggregate labor share which we focus on below.²²

38 The top panel of Figure 5 depicts the observed (HP-filtered) fluctuations in the labor share
 39 together with those predicted by the benchmark model. While the model-predicted labor share is
 40 somewhat more volatile, as discussed earlier, the model does very well in capturing the business
 41 cycle fluctuations of the labor share throughout the sample period (the correlation coefficient
 42 between the two time series is 0.55).

¹⁹Recently, Gertler et al. (2016) find that distinguishing job-to-job flows from unemployment hires may be important. While the presented model abstracts from job-to-job flows, it would be interesting to analyze to what extent the observed labor share dynamics are driven by wage changes of different groups of workers.

²⁰See the Appendix for details on the estimation procedure.

²¹Depending on how wages and productivity are measured, estimates of η_{all} range between 0.19 and 0.43, while estimates of η_{new} vary between 0.54 and 1.07 (see Haefke et al., 2013, for detailed empirical robustness exercises).

²²The Appendix provides further details on the estimation and shows that the time paths of other model variables, not used in the estimation, closely follow those observed in the data.

1 *4.4.1. Switching off competition and cohort effects*

2 One of the key features of our model is the persistent effect on both production and wages of
 3 *the degree of firm competition at the time of hiring*. This single feature leads to both “competition
 4 effects” and “cohort effects”. To illustrate the importance of these novel aspects of our model in
 5 accounting for the dynamics of the labor share, we consider what happens when this key feature
 6 is “switched off”.

7 In particular, we consider a “counter-factual” alternative model which is not micro-founded,
 8 but which retains the steady state equilibrium conditions of our benchmark economy. Specifically,
 9 we fix the permanent component of match productivity for all matches to its steady state value,
 10 $p(\theta_{ss})$. This eliminates the effect on production of firm competition at the time of hiring. We also
 11 fix the share of bilateral matches – a measure of the degree of firm competition – to its steady
 12 state value, $\mu(\theta_{ss})$, and assume that the wage paid in bilateral matches is updated each period.
 13 These assumptions eliminate the persistent effect on wages of firm competition at the time of
 14 hiring, while keeping the basic wage structure intact. The rest of the benchmark model, including
 15 all equilibrium conditions and aggregate shocks, remains the same.

16 This counter-factual model is similar in *spirit* to standard Diamond-Mortensen-Pissarides
 17 (DMP) model. In such models, there is no dependence on the degree of firm competition at
 18 the time of hiring and therefore no “competition” or “cohort” effects. In such models, the share of
 19 bilateral meetings is constant (since all meetings are bilateral) and the average match productivity
 20 does not vary with the market tightness θ_t . Instead, the output of all employment relationships
 21 fluctuates only with aggregate TFP shocks, and wages depend only on *current* labor market
 22 conditions (and TFP).

23 All of these DMP-style features are captured in the counter-factual model, while retaining all
 24 the equilibrium conditions of our benchmark economy, including its basic wage structure. This
 25 experiment therefore gives us a sense of how the competition and cohort effects – key to this
 26 model but absent from standard DMP models – play a crucial role in accounting for the cyclical
 27 fluctuations in the labor share.

28 In the counter-factual model, the aggregate labor share $s_{L,t}^c$ can be expressed as:

$$s_{L,t}^c = 1 - \lambda - \left(1 - \frac{b_t^c}{A_t}\right) \frac{\mu(\theta_{ss})}{p(\theta_{ss})}, \quad (41)$$

29 where b_t^c is the counter-factual wage paid in bilateral meetings at time t .²³ Note that there is no
 30 distinction between new and existing hires. All “cohorts” face the same wages and labor share
 31 (i.e. no cohort effects). Also, neither the average match productivity nor the share of bilateral
 32 meetings fluctuate over the business cycle (i.e. no competition effects). The aggregate labor share
 33 still fluctuates, however, since it directly depends on aggregate TFP shocks A_t and on fluctuations
 34 in the counter-factual reservation wage b_t^c . The latter varies over time because it depends on the
 35 values of unemployment and employment, and the job finding probability, all of which fluctuate
 36 endogenously in the counter-factual economy.

37 In order to quantify the importance of firm competition at the time of hiring for labor share
 38 fluctuations, we redo the above exercise using the counter-factual model which no longer features
 39 competition and cohort effects. In particular, we use the counter-factual model to again back
 40 out the underlying aggregate TFP and investment-specific shocks using data on real GDP and
 41 unemployment. The labor share implied by the counter-factual model is depicted in the bottom
 42 panel of Figure 5.

43 In contrast to the benchmark economy, the counter-factual model fails to account for the
 44 cyclical fluctuations in the labor share. In fact, the labor share predicted by the counter-factual
 45 model is essentially uncorrelated with its empirical counterpart (the correlation coefficient is 0.01).
 46 Therefore, the novel features of our model – namely, the direct competition between firms to hire
 47 workers and the resulting cohort effects – are crucial for understanding the cyclical fluctuations
 48 of the U.S. labor share, both qualitatively and quantitatively.

²³In equation (41), we have $x_{0t} = 1$, since under our calibration the cut-off productivity for firm participation x_t^c is always below $x_{\min} = 1$.

5. Conclusion

While the relative constancy of factor shares has been a stylized fact in macroeconomics for decades, factor shares are not constant in the data. Importantly, movements in the labor share systematically respond to TFP shocks and such movements have important implications for aggregate dynamics (Rios-Rull and Santaaulalia-Llopis, 2010). This paper contributes to our understanding of the business cycle properties of the labor share. We develop a tractable dynamic search and matching model in which heterogeneous firms compete to hire workers. Such competition endogenizes both the aggregate production technology and the division of output between workers and firms. We show that the model can account well for both the counter-cyclicality of the U.S. labor share and its overshooting pattern following TFP shocks, as well as its time path.

While the focus of this paper is on understanding the labor share, the model makes a host of other predictions regarding important issues in macroeconomics. For example, how do fluctuations in the degree of firm productivity dispersion affect labor productivity, unemployment, and the wage distribution? The model provides an integrated framework for addressing such questions. Moreover, the vintage structure of the model provides a way of studying cohort effects. In particular, it directs attention to the *composition* of the employment pool with respect to labor market conditions at the time of hiring. Given the existing evidence on firm and worker cohort effects, it would be interesting to use the model to examine the persistent scarring effects of entering the labor market during a downturn. These questions are left for future research.

References

- Bentolila, S., Saint-Paul, G., 2003. Explaining Movements in the Labor Share. *Contributions to Macroeconomics* 3 (1).
- Bils, M., 1985. Real wages over the business cycle: Evidence from panel data. *Journal of Political Economy* 93 (4), 1–76.
- Blanchard, O. J., 1997. The Medium Run. *Brookings Papers on Economic Activity* 2, 89–158.
- Canova, F., Lopez-Salido, D., Michelacci, C., 2013. The Ins and Outs of Unemployment: An Analysis Conditional on Technology Shocks. *Economic Journal* 123.
- Choi, S., Rios-Rull, J.-V., 2009. Understanding the Dynamics of Labor Share: The Role of Noncompetitive Factor Prices. *Annals of Economics and Statistics* (95/96), 251–277.
- Colciago, A., Rossi, L., 2015. Firm Dynamics, Endogenous Markups, and the Labor Share of Income. *Macroeconomic Dynamics* 19, 1309–1331.
- Elsby, M., Michaels, R., 2013. Marginal jobs, heterogeneous firms and unemployment flows. *American Economic Journal: Macroeconomics* 5 (1), 1 – 48.
- Elsby, M. W., Hobijn, B., Şahin, A., 2013. The decline of the us labor share. *Brookings Papers on Economic Activity* 2013 (2), 1–63.
- Fisher, J., 2006. The Dynamic Effects of Neutral and Investment-Specific Technology Shocks. *Journal of Political Economy* 114 (3), 413–451.
- Gertler, M., Huckfeldt, C., Trigari, A., 2016. Unemployment Fluctuations, Match Quality, and the Wage Cyclicity of New Hires.
- Gomme, P., Greenwood, J., 1995. On the cyclical allocation of risk. *Journal of Economic Dynamics and Control* 19, 91–124.
- Haefke, C., Sonntag, M., van Rens, T., 2013. Wage Rigidity and Job Creation. *Journal of Monetary Economics* 60 (8), 887–899.
- Jones, C. I., 2005. The Shape of Production Functions and the Direction of Technical Change. *Quarterly Journal of Economics* 120 (2), 517–549.
- Jung, P., Kuester, K., 2015. Optimal Labor Market Policy in Recessions. *American Economic Journal: Macroeconomics* 7 (2), 124–156.
- Justiniano, A., Primiceri, G., 2008. The Time Varying Volatility of Macroeconomic Fluctuations. *American Economic Review* 98 (3), 604–641.
- Karabarbounis, L., Neiman, B., 2014. The Global Decline of the Labor Share. *Quarterly Journal of Economics* 129 (1), 61–103.
- King, R. G., Watson, M. W., 2012. Inflation and unit labor cost. *Journal of Money, Credit and Banking* 44 (s2), 111–149.
- Lagos, R., 2006. A Model of TFP. *Review of Economic Studies* 73 (4), 983–1007.
- Lopez-Salido, D., Michelacci, C., 2007. Technology shocks and job flows. *Review of Economic Studies* 74 (4), 1195–1227.

- 1 Mangin, S., 2017. A Theory of Production, Matching, and Distribution. *Journal of Economic Theory* (forthcoming).
- 2 Oreopoulos, P., Heisz, A., von Wachter, T., 2012. The Short- and Long-Term Career Effects of Graduating in a
3 Recession. *American Economic Journal: Applied Economics* 4 (1), 1–29.
- 4 Piketty, T., 2014. *Capital in the Twenty-First Century*. Belknap Press, Cambridge, Massachusetts.
- 5 Piketty, T., Zucman, G., 2014. Capital is back: Wealth-income ratios in rich countries 1700-2010. *Quarterly Journal*
6 *of Economics* 129 (3), 1255–1310.
- 7 Rios-Rull, J.-V., Santaaulalia-Llopis, R., 2010. Redistributive Shocks and Productivity Shocks. *Journal of Monetary*
8 *Economics* 57 (8), 931–948.
- 9 Sedláček, P., Sterk, V., 2017. The Growth Potential of Startups over the Business Cycle. *American Economic*
10 *Review* 107 (10), 3182–3210.
- 11 Shimer, R., 2005. The Cyclical Behavior of Equilibrium Unemployment and Vacancies. *American Economic Review*
12 95 (1), 25–49.
- 13 Syverson, C., 2004. Product Substitutability and Productivity Dispersion. *Review of Economics and Statistics*
14 86 (2), 534–550.

Table 1: Calibrated parameters and their respective targets

parameter	value	target/source
<i>household and labor market</i>		
β	discount factor	0.996 annual interest rate 4%
z	value of non-market activity	0.320 labor share 68%, RS (2010)
δ	separation rate	0.050 unemployment rate 5.8%, BLS
\bar{C}	vacancy posting cost	6.361 job finding rate 45%, Shimer (2005)
<i>firm productivity distribution $G(x)$</i>		
x_{\min}	minimum	1 normalization
λ	shape parameter	0.27 90 – 10 TFP range 0.83, Syverson (2004)
<i>aggregate shocks</i>		
ρ_A	TFP, persistence	0.931 output persistence, BEA
σ_A	TFP, standard deviation	0.007 output volatility, BEA
ρ_C	investment-specific, persistence	0.906 labor productivity persistence, BLS
σ_C	investment-specific, volatility	0.033 labor productivity volatility, BLS

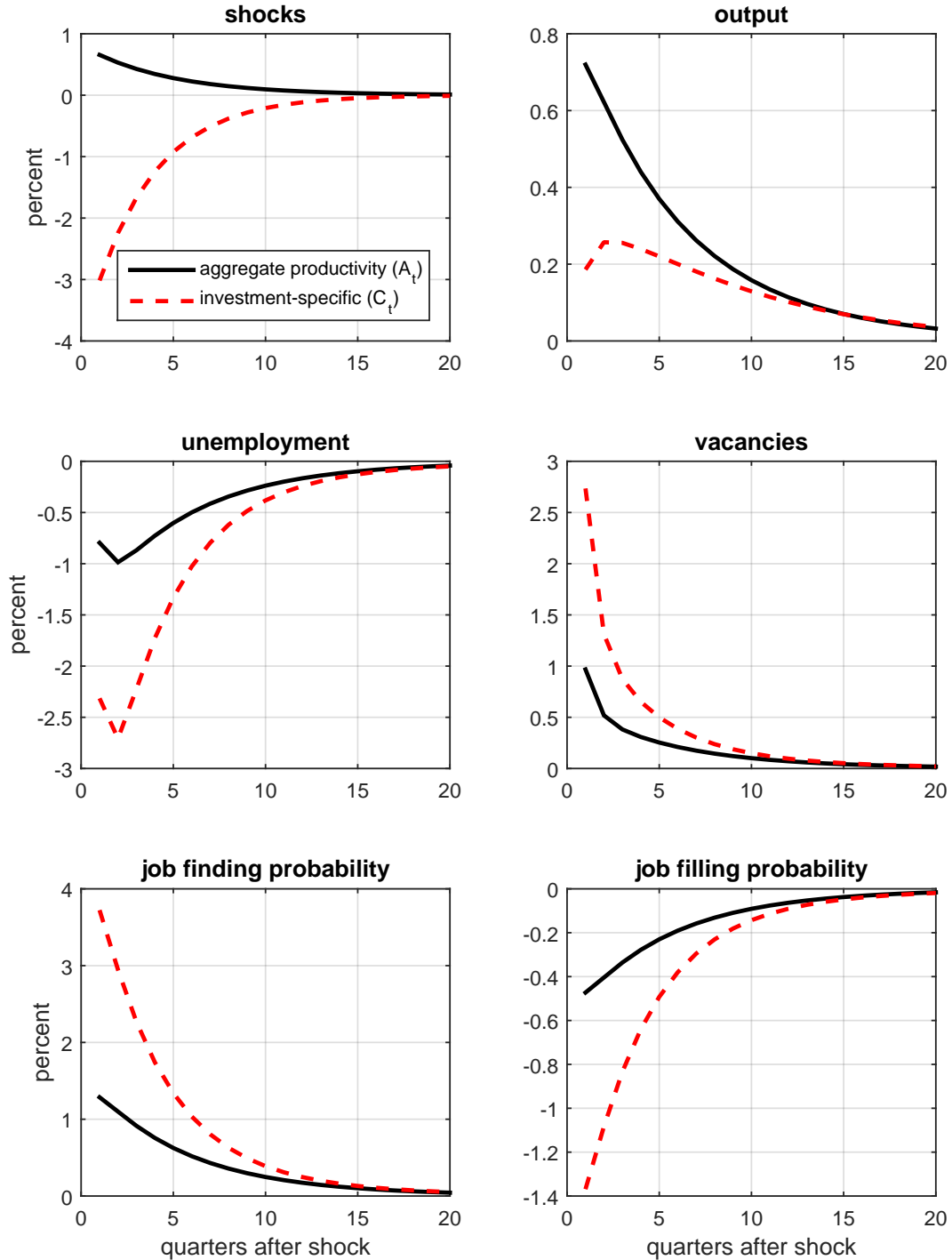
Notes: The table reports model parameters and their respective targets or sources. “BLS” and “BEA” stand for, respectively, the Bureau of Labor Statistics and the Bureau of Economic Analysis. “RS (2010)” refers to Rios-Rull and Santaaulalia-Llopis (2010). Output and labor productivity are measured, respectively, as real GDP and as output per worker in the non-farm business sector.

Table 2: Business cycle statistics

	U.S. data				model				
	Y	u	θu	m	Y	u	θu	m	
relative volatility	1.2	8.7	10.2	5.7	1.3	5.6	4.1	6.7	
correlations	Y	1	-0.83	0.89	0.82	1	-0.63	0.51	0.61
	u		1	-0.91	-0.91		1	-0.81	-0.96
	θu			1	0.91			1	0.93

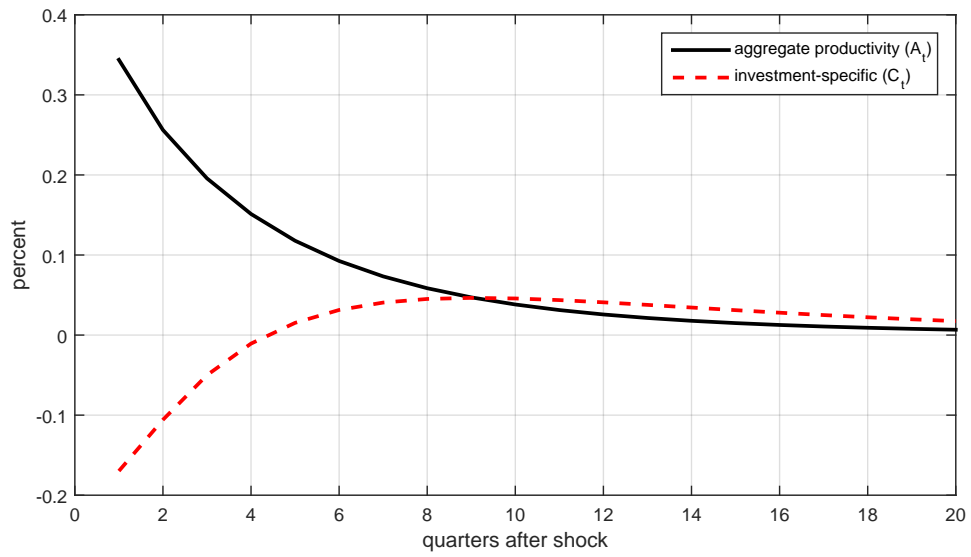
Notes: The table reports the standard deviations of labor market variables relative to that of labor productivity (“relative volatility”) and the associated correlation matrix both for the U.S. data and the model. The sample period considered runs from 1954Q1 to 2004Q4. Y refers to real GDP, u is the number of unemployed, θu is the number of vacancies, and m is the job finding probability. All data are taken from Rios-Rull and Santaaulalia-Llopis (2010), except for the job finding probability, which is taken from Shimer (2005). The data are logged and HP filtered with a smoothing coefficient of 1600. The simulated data from the model is treated in the same way and has the same sample period length.

Figure 1: Impulse response functions of labor market variables



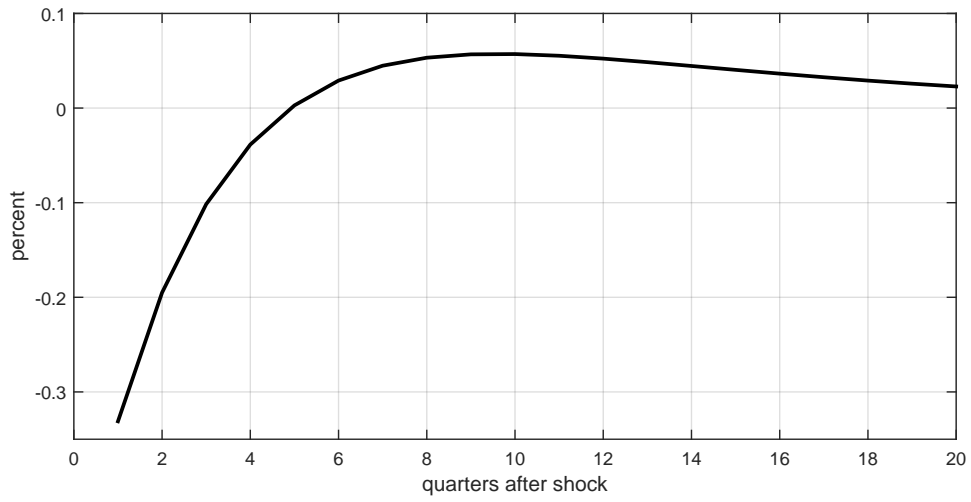
Notes: Impulse response functions to expansionary one-standard-deviation shocks to aggregate TFP (black solid line) and investment-specific technology (red dashed line). All variables are expressed in percent deviations from their respective steady state values.

Figure 2: Impulse response function of labor productivity



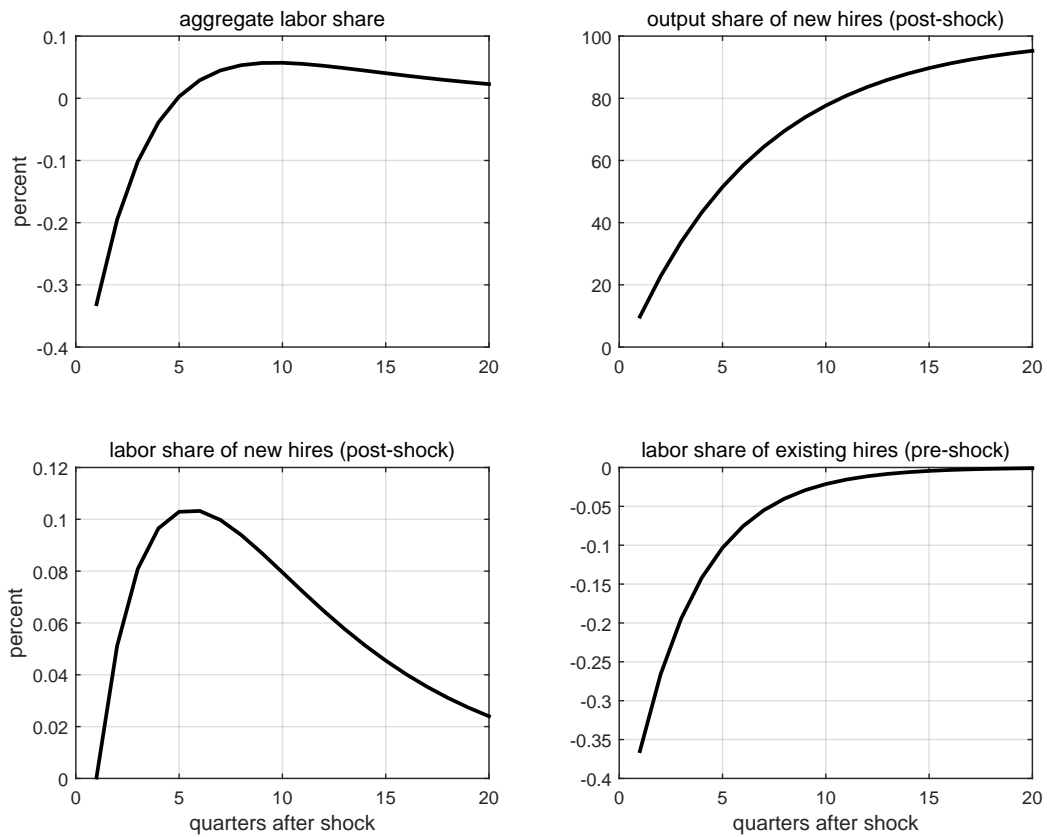
Notes: Impulse response function to expansionary one-standard-deviation shocks to aggregate TFP (black solid line) and investment-specific technology (red dashed line). Labor productivity is expressed in percent deviations from its steady state values.

Figure 3: Impulse response function of the labor share



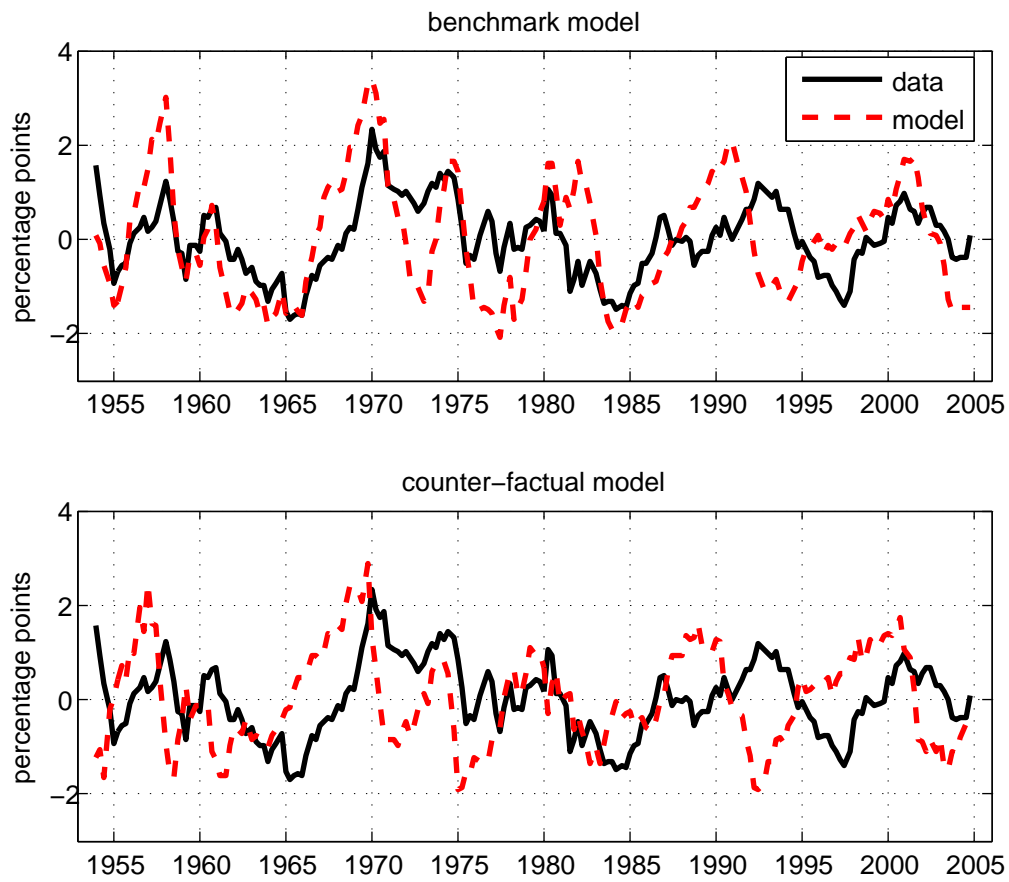
Notes: Impulse response function to an expansionary one-standard-deviation shock to aggregate TFP expressed as percent deviations from steady state.

Figure 4: Impulse response function for new hires and existing matches



Notes: Impulse response function to an expansionary one-standard-deviation shock to aggregate TFP expressed as percent deviations from steady state. “Labor share of new hires (post-shock)” and “labor share of existing hires (pre-shock)” refer to, respectively, the average wage-to-output ratios in employment relationships formed after and before the aggregate TFP shock hits the economy. “Employment share of new hires (post-shock)” refers to the share of employment relationships formed after the aggregate TFP shock.

Figure 5: Time path of the labor share: data and model



Notes: The labor share data is taken from Rios-Rull and Santaaulalia-Llopis (2010), HP-filtered. The top panel shows the model-predicted labor share in the benchmark model and the bottom panel shows the counter-factual model-predicted labor share without competition and cohort effects. Both model-predicted time-series are constructed based on aggregate TFP and investment-specific shocks backed out from the respective models using the Kalman filter and data on real GDP and the unemployment rate. All variables are expressed in percentage point deviations from their respective trends.