Competitive Dispersion, Price Dispersion, and Markups

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Abstract

Equilibrium price dispersion for identical goods can arise from dispersion in the effective competition for each buyer, i.e. dispersion in the number of price quotes a buyer obtains or in the number of sellers in a buyer's choice set. We call this *competitive dispersion*. This paper studies the effects of competitive dispersion on equilibrium markups and price dispersion in a Burdett-Judd search-theoretic model of imperfect competition. We find that greater competitive dispersion can either increase or decrease the aggregate markup, depending on the degree of competition. Surprisingly, greater competitive dispersion does not always increase price dispersion. Equilibrium markups and price dispersion are constrained efficient if and only if the search technology is Poisson, which minimizes competitive dispersion but not necessarily price dispersion. We apply our results to the labor market and find that greater competitive dispersion can either increase or decrease both the average wage and wage dispersion.

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1 Introduction

Price dispersion for identical goods arises in many markets. Perhaps more surprisingly, price dispersion can emerge as an equilibrium outcome even when both buyers and sellers are ex ante homogeneous. For example, consider the well-known Burdett and Judd (1983) search-theoretic model of imperfect competition.[1] In this model, equilibrium price dispersion can arise even for homogeneous firms selling identical goods to ex ante identical buyers. This model features in many recent papers, such as Menzio (2023, 2024a,b), Albrecht, Menzio, and Vroman (2023), Shi (2023), Nord (2023), Pytka (2024), Mangin and Menzio (2024), Chernoff, Head, and Lapham (2024, 2025), and Sangani (2025), and has been used to study not only price dispersion but labor markets, monetary policy, sticky prices, pass-through, and market power.

In the Burdett-Judd model, price dispersion arises as a result of dispersion in the degree of effective competition for buyers, which may be due to search or other frictions. Dispersion in effective competition for a buyer may be interpreted as dispersion in the number of price quotes a buyer obtains, the number of sellers a buyer finds or "meets," or the size of a buyer's choice set. We call this *competitive dispersion*.

Competitive dispersion is necessary for price dispersion with homogeneous sellers. If all buyers are "captive" (i.e. obtain only one quote), there is a single price: the monopoly price. If all buyers are "non-captive" (i.e. obtain two or more quotes), there is a single price: the competitive price. When there is competitive dispersion (i.e. some buyers are captive and some are non-captive), there is price dispersion.

This paper studies the effects of competitive dispersion on equilibrium outcomes in the Burdett-Judd model. How does competitive dispersion affect both markups and price dispersion? Does greater competitive dispersion raise or lower the aggregate markup? Does more competitive dispersion always lead to more price dispersion?

Despite the large literature on the Burdett-Judd model, these questions have not yet been answered – or even asked – because the standard distributions used are the two-point distribution (i.e. buyers receive either one or two quotes) and the Poisson distribution. In both cases, we cannot separate the effects of the *degree of competition* (i.e. the average number of quotes a buyer receives) from the effects of *competitive dispersion* (i.e. the dispersion of the distribution of the number of price quotes).

[1] By the "Burdett-Judd model", we mean the non-sequential search model that can be attributed to Butters (1977), Varian (1980), and Burdett and Judd (1983).

To answer the above questions, we need to move beyond these two distributions. We consider a richer class of search technologies that determine the distribution of the size of a buyer's choice set. In particular, we focus on the class of invariant search technologies introduced in Lester, Visschers, and Wolthoff (2015), which has been recently used in Cai, Gautier, and Wolthoff (2025) and Mangin (2026).

The class of invariant search technologies is useful for a number of reasons. First, it delivers efficiency of seller entry in the Burdett-Judd model, which is otherwise not guaranteed. Second, it allows us to obtain tractable and intuitive closed-form solutions for all equilibrium outcomes. Third, we can recover the standard results for the Poisson benchmark as a special limiting case. Fourth, by considering a two-parameter family within this class, we can cleanly separate the effect of changes in the *degree of competition* from the effect of changes in *competitive dispersion*, which is not possible for the Poisson. Finally, we discuss the fact that there is a one-to-one mapping from *any* distribution of competitive intensity to a search technology in the invariant class, as shown in Cai et al. (2025). This allows the Burdett-Judd framework with invariant search technologies to incorporate buyer heterogeneity (e.g. heterogeneity in search effort or other types of heterogeneity such as geographic location or income) in a tractable manner that is consistent with our aggregate results, and to consider how outcomes vary across buyers of different types (e.g. search effort or income).

Main results. First, we show that the assumption of invariance of the search technology greatly simplifies the equilibrium outcomes in the Burdett-Judd model. The distribution of posted prices, the equilibrium pricing condition, the quantile markup function, and the quantity demanded can all be expressed solely in terms of the function which determines the probability a seller has no competitors.

Second, we find that greater competitive dispersion can either increase or decrease the aggregate markup depending on both the search technology and the degree of competition. We consider the negative binomial family, which nests the Poisson as a limiting case. We show that in less competitive markets, greater competitive dispersion decreases the aggregate markup. In more competitive markets, the effect is non-monotonic. For lower levels of competitive dispersion, the aggregate markup is increasing in competitive dispersion, but for higher levels of competitive dispersion, the aggregate markup is decreasing in competitive dispersion.

Third, even though competitive dispersion is necessary for equilibrium price dispersion, we find that an increase in competitive dispersion does *not* necessarily lead to higher price dispersion. Whether or not greater competitive dispersion increases price dispersion depends on both the search technology and the degree of competition.

Fourth, we model seller entry and show that constrained efficiency of seller entry does not always hold outside the special case of the Poisson distribution. We derive a precise condition that is necessary for constrained efficiency of seller entry, which is satisfied by the class of invariant search technologies. This is consistent with the more general result on constrained efficiency established in Lester et al. (2015).

Fifth, when we allow the planner to choose the degree of competitive dispersion, we find that the Poisson search technology is uniquely socially optimal within the invariant class. The Poisson search technology minimizes competitive dispersion but not necessarily price dispersion: higher price dispersion is sometimes socially optimal. We can interpret excess competitive dispersion relative to the Poisson as a kind of "misallocation" of competition across buyers which reduces welfare. We provide a numerical example to illustrate the welfare loss from competitive dispersion.

Finally, we consider a simple application of our results to the labor market. In the labor market, competitive dispersion represents dispersion in the degree of effective competition to hire workers. We find that greater competitive dispersion always leads to higher unemployment, but it can either increase or decrease the average wage, depending on both the search technology and the labor market tightness. Greater competitive dispersion may either increase or decrease wage dispersion, depending on the search technology, the labor market tightness, and workers' outside option.

Related literature. Menzio (2024b) examines how equilibrium markups in the Burdett-Judd model depend on the *degree of competition*, i.e. the average size of buyers' choice sets, as well as on the distribution of marginal costs across heterogeneous firms. Menzio (2024b) uses the Poisson distribution, so it is not possible to consider the effects of *competitive dispersion*. Complementary to that paper, we focus on the case of homogeneous firms but consider a richer class of search technologies. This enables us to provide some general theoretical results regarding how both the mean and the dispersion of the distribution of the size of buyers' choice sets affects markups and price dispersion. In addition, we show that the efficiency result in Menzio (2024b) depends on the search technology and does not always hold outside the Poisson. This allows us to consider the welfare effects of greater competitive dispersion.

There is a large literature on the Burdett-Judd model and its applications. However, the majority of papers focus on either the two-point distribution or the Poisson distribution, which do not allow us to answer the questions we consider. The two-point distribution is studied in the original Burdett and Judd (1983), as well as in later applications and variations of the Burdett-Judd model, such as Head, Liu, Menzio, and Wright (2012), Kaplan and Menzio (2016), Menzio and Trachter (2018), Lester, Shourideh, Venkateswaran, and Zetlin-Jones (2019), Kaplan, Menzio, Rudanko, and Trachter (2019), Nord (2023), Pytka (2024), and Menzio (2024a). The Poisson distribution is used in Mortensen (2005), as well as later applications of the Burdett-Judd model such as Burdett, Trejos, and Wright (2017), Menzio (2023), Albrecht et al. (2023), Shi (2023), Mangin and Menzio (2024), Menzio (2024b), and Sangani (2025).

Some papers in the literature allow for more general distributions, including Head and Kumar (2005), Head, Kumar, and Lapham (2010), and later papers by Herrenbrueck (2017), Chernoff et al. (2024, 2025), and Sangani (2025). While these papers consider general distributions, they specialize to specific distributions for their results (either by restriction or by endogenizing the distribution) and focus on applications such as monetary economics, cost pass-through, and price dispersion across the income distribution, which are complementary to the theoretical focus of this paper.

Our focus on invariant search technologies complements the existing literature regarding this class of search technologies, which was introduced in Lester et al. (2015). That paper derived an important efficiency result: the optimality of competing auctions with a reservation price equal to the sellers' outside option requires invariance of the search technology. Our efficiency result is consistent with their result, although we consider a Burdett-Judd environment with homogeneous firms and we derive our sufficient condition in a simple way. A recent paper by Cai et al. (2025) shows that the class of invariant search technologies is equivalent to the class of mixed Poisson search technologies, a result that is used extensively in Mangin (2026) to study extreme value theory with heterogeneous agents using this class of search technologies.

Outline. Section 2 presents the environment. Section 3 characterizes equilibrium under both general and invariant search technologies. Section 4 studies how equilibrium outcomes vary with the degree of competition. Section 5 isolates the effects of competitive dispersion on equilibrium outcomes. Section 6 considers entry and constrained efficiency, and examines the welfare implications of competitive dispersion. Section 7 provides a microfoundation via buyer heterogeneity in competitive intensity. Section 8 applies the framework to the labor market and examines its implications. All of the proofs omitted from the main text can be found in the Appendix.

2 Environment

There is a continuum of measure b of ex ante identical buyers and a continuum of measure s of ex ante identical sellers. Each buyer wants to buy one unit of the good and each seller wants to sell one unit of the good, which they can produce at marginal cost $c \geq 0$. All agents are risk-neutral.

All buyers have the same valuation $v > c$ for the good, which is identical across sellers. For simplicity, we assume that both sellers and buyers have no outside option so the trade surplus is $v - c$. A buyer's payoff from purchasing the good at price $p > 0$ is $v - p$, and a seller's payoff from a sale is $p - c$. If the probability of a sale is q, a seller's expected payoff is $q(p - c)$. We can interpret q as the *quantity* demanded.[2]

Let $\lambda = s/b$ denote the ratio of sellers to buyers. We assume that the average number of sellers a buyer meets is λ. We refer to λ as the *degree of competition*.

We say that buyers "meet" $k \in \mathbb{N} = \{0, 1, 2, ...\}$ sellers if they obtain a price quote from k sellers, i.e. have k sellers in their choice set. The number of sellers a buyer meets is a random variable that is given by a *search technology* $\{P_k\}_{k=0}^{\infty}$, which we denote simply by P_k. For any given λ, the search technology P_k determines the probability mass function, $P_k(\lambda) : \mathbb{N} \to [0, 1]$ where $P_k(\lambda)$ denotes the probability that a buyer meets k sellers given λ. We assume $\sum_{k=0}^{\infty} P_k(\lambda) = 1$ and $\sum_{k=0}^{\infty} kP_k(\lambda) = \lambda$.

It is useful to consider the distribution $P_k(\lambda)$ from the perspective of sellers. Let $Q_k(\lambda)$ denote the probability a seller is in a meeting with $k \geq 1$ sellers (including that seller). Equivalently, $Q_k(\lambda)$ is the probability that a seller has $k-1$ competitors. The distribution $Q_k(\lambda)$ is implied by the distribution $P_k(\lambda)$, which is exogenous. An important identity, which allows us to switch between these two distributions, is

$$\lambda Q_k(\lambda) = kP_k(\lambda). \tag{1}$$

The above identity must hold for all $k \geq 1$ to ensure that P_k and Q_k are consistent.[3]

We assume that trades always go through if buyers and sellers are indifferent. Therefore, buyers always purchase if they meet at least one seller. The probability of

[2] Alternatively, we could assume there is a continuum of measure one of buyers per seller, in which case q would indeed be the *quantity* of units sold rather than the *probability* of a sale. All of the results and expressions in the paper would be identical under this assumption.

[3] See Eeckhout and Kircher (2010) and Lester et al. (2015). To see why this identity must hold, the measure of sellers which have $k - 1$ competitors is $sQ_k(\lambda)$ and this must be equal to $kbP_k(\lambda)$, which is k times the measure of buyers who meet k sellers, which implies (1).

purchase for a buyer is therefore the probability of matching for a buyer, which is

$$m(\lambda) \equiv 1 - P_0(\lambda). \tag{2}$$

We assume that P_0 is at least twice continuously differentiable and that $P_0'(\lambda) \leq 0$, $P_0''(\lambda) \geq 0$, and $\lim_{\lambda\to\infty} P_0(\lambda) = 0$, so $m'(\lambda) \geq 0$, $m''(\lambda) \leq 0$, and $\lim_{\lambda\to\infty} m(\lambda) = 1$.

Sellers post prices and buyers purchase from the seller they meet that offers the highest surplus $v - p$, provided that the surplus is weakly positive. Buyers always choose to purchase from the lowest-price seller they meet since buyers value the good offered by all sellers at v. The seller-symmetric mixed-strategy equilibrium is a distribution of posted prices, each of which maximizes a seller's expected profit.

3 Equilibrium

Burdett and Judd (1983) show that competitive dispersion is necessary for equilibrium price dispersion. In the case where all buyers meet only one seller (i.e. all buyers are captive), there is no price dispersion because the posted price is $p = v$ for all sellers. In the case where all buyers meet two sellers (i.e. all buyers are non-captive), there is no price dispersion because the posted price is $p = c$ for all sellers.

We focus on the interesting case where there is a positive probability of both captive buyers (who meet one seller) and non-captive buyers (who meet two or more sellers). We assume $P_1(\lambda) > 0$ and $P_k(\lambda) > 0$ for some $k \geq 2$. As shown in Burdett and Judd (1983), the equilibrium distribution of posted prices is a nondegenerate, atomless, and continuous distribution that represents firms' mixed strategies, which are symmetric in the sense that all firms draw prices from the *same* distribution.

3.1 General search technology

We first characterize equilibrium in the Burdett-Judd model for a general search technology. Propositions 1 and 2 are not novel but are important for understanding the model before we consider invariant search technologies in Section 3.2.

Distribution of posted prices. Proposition 1 derives the equilibrium distribution of posted prices. This result is derived in Burdett and Judd (1983) and more recently in Menzio (2024b) for the Poisson search technology.

Proposition 1. *For any degree of competition λ, there is a unique seller-symmetric mixed-strategy equilibrium. The equilibrium distribution of posted prices has no mass points and cdf F with support $[\underline{p}, v]$ where $\underline{p} = c + Q_1(\lambda)(v - c)$.*

The equilibrium distribution of posted prices F is implicitly determined by:

$$p = c + \frac{Q_1(\lambda)}{q(p)}(v - c) \tag{3}$$

where $q(p)$ is the quantity demanded at posted price p:

$$q(p) = \sum_{k=1}^{\infty} Q_k(\lambda)(1 - F(p))^{k-1} \tag{4}$$

and $Q_k(\lambda)$ is the probability a seller is one of k sellers that meets a buyer.

The equilibrium is a seller-symmetric mixed strategy equilibrium in the sense that all sellers choose the *same* mixed strategy, which is essentially to draw prices randomly from the distribution F. In equilibrium, all firms make equal expected profits, so $\pi(p) = (p - c)q(p) = Q_1(\lambda)(v - c)$ for *all* posted prices, and equation (3) reflects this fact. Equation (4) says that the quantity demanded at price p is the probability that every one of a seller's $k-1$ competitors is a higher-priced competitor.

Firms are indifferent between prices p on the support of F because an increase in the net price $p-c$ is exactly offset by a decrease in the probability of sale or quantity demanded. Intuitively, some firms charge higher prices and face lower demand, while other firms charge lower prices and face higher demand, but profits are the same.

Distribution of transaction prices. The distribution of *transaction prices* H is the distribution of prices actually paid by buyers to their choice of seller. This is different from the distribution of *posted prices* F because buyers only purchase from the seller they meet with the lowest price. Both distributions have the same support.

Let $\tilde{q}(\lambda) = m(\lambda)/\lambda$, the *average* quantity demanded for a seller in the market.[4]

Lemma 1. *For any degree of competition λ, the distribution of transaction prices has support $p \in [\underline{p}, v]$ and the cdf H of the distribution of transaction prices is*

$$H(p) = \frac{\sum_{k=1}^{\infty} P_k(\lambda)[1 - (1 - F(p))^k]}{1 - P_0(\lambda)}. \tag{5}$$

[4]The total expected number of sales is $m(\lambda)b$, which must equal $\tilde{q}(\lambda)s$, so $\tilde{q}(\lambda) = m(\lambda)/\lambda$.

The density h of the distribution of transaction prices is

$$h(p) = \frac{q(p)}{\tilde{q}(\lambda)} f(p). \tag{6}$$

Proof. Conditional on k sellers, the transaction price is the minimum of k i.i.d. draws from F, so $\Pr(X \leq p \mid K = k) = 1-(1-F(p))^k$. Taking the expectation with regard to the distribution P_k and conditioning on $K \geq 1$ yields expression (5) for H.

Next, we differentiate H to get $h = H'$. Using expression (4) for $q(p)$ and the fact that $\tilde{q}(\lambda) = m(\lambda)/\lambda$ where $m(\lambda) = 1 - P_0(\lambda)$, we obtain expression (6) for $h(p)$. □

Aggregate price. Given the distribution of transaction prices H, the average transaction price $\tilde{p}(\lambda) \equiv \mathbb{E}_H[p]$ can be expressed as follows. Starting with expression (6) for the density h, we can write the following:

$$\tilde{p}(\lambda) \equiv \mathbb{E}_H[p] = \int ph(p)dp = \int p \frac{q(p)}{\tilde{q}(\lambda)} f(p)dp. \tag{7}$$

The average transaction price is therefore the quantity-weighted average of the prices posted by sellers. In this way, $\tilde{p}(\lambda)$ is indeed an *aggregate price* in the usual sense.

Proposition 2 derives a general expression for the aggregate price $\tilde{p}(\lambda)$. To express this result, let $\alpha(\lambda)$ denote the proportion of sales that are *competitive*, i.e. sales that are made when a seller has at least one competitor, and let $1 - \alpha(\lambda)$ denote the proportion of monopoly sales, i.e. sales that are made with no competitors.

Proposition 2. *For any degree of competition λ, the aggregate price is*

$$\tilde{p}(\lambda) = c + (1 - \alpha(\lambda))(v - c) \tag{8}$$

where $1 - \alpha(\lambda) = Q_1(\lambda)/\tilde{q}(\lambda)$ is the proportion of monopoly sales.

Observe that the aggregate price can be expressed as a weighted average of the monopoly price v and the competitive price c, weighted by the proportion of sales:

$$\tilde{p}(\lambda) = (1 - \alpha(\lambda))v + \alpha(\lambda)c. \tag{9}$$

In the limit as $\alpha(\lambda) \to 0$, we obtain the monopoly price v. In the limit as $\alpha(\lambda) \to 1$, we obtain the competitive price c. We discuss these limits further in Section 4.3.

Given the aggregate price, the equilibrium pricing condition (3) implies the following relationship between relative quantities and relative (net) prices:

$$\frac{q(p)}{\tilde{q}(\lambda)} = \left(\frac{p-c}{\tilde{p}(\lambda)-c}\right)^{-1}. \tag{10}$$

If a seller sets a lower (higher) price than average ($p < \tilde{p}(\lambda)$), the quantity demanded is higher (lower) than the average ($q(p) > \tilde{q}(\lambda)$). If a seller sets the average price $\tilde{p}(\lambda)$, the quantity demanded is average; that is, we have $q(\tilde{p}(\lambda)) = \tilde{q}(\lambda)$.

Price dispersion. Our primary measure of price dispersion is the coefficient of variation $cv_H(\lambda)$ of the distribution H of transaction prices, which is defined by $cv_H^2(\lambda) \equiv \sigma_H^2(\lambda)/\tilde{p}(\lambda)^2$ where $\sigma_H^2(\lambda)$ is the variance of H. The coefficient of variation $cv_H(\lambda)$ is a measure of price dispersion *relative to the mean* and is sometimes called the "normalized standard deviation". Proposition 3 derives a general expression.

Proposition 3. *For any degree of competition λ, equilibrium price dispersion satisfies*

$$cv_H^2(\lambda) = \left(\frac{\mathbb{E}_F\left[p-c\right]}{\tilde{p}(\lambda)-c} - 1\right)\left(\frac{\tilde{p}(\lambda)-c}{\tilde{p}(\lambda)}\right)^2. \tag{11}$$

In the limit as marginal cost $c \to 0$, equilibrium price dispersion satisfies

$$cv_H^2(\lambda) = \frac{\mathbb{E}_F\left[p\right]}{\tilde{p}(\lambda)} - 1. \tag{12}$$

Observe that the aggregate net price can be written as a weighted harmonic mean:

$$\tilde{p}(\lambda) - c = \left(\mathbb{E}_F[(p-c)^{-1}]\right)^{-1} \tag{13}$$

This is because the quantity demanded $q(p)$ is inversely proportional to the net price $p - c$ and the aggregate price is a quantity-weighted average of posted prices.

The first term in our price dispersion expression (11) is a ratio of a mean to a harmonic mean. This ratio is an intuitive measure of dispersion:

$$\frac{\mathbb{E}_F[p-c]}{\tilde{p}(\lambda)-c} = \frac{\mathbb{E}_F[p-c]}{(\mathbb{E}_F[(p-c)^{-1}])^{-1}} = \mathbb{E}_F[p-c]\mathbb{E}_F[(p-c)^{-1}]. \tag{14}$$

This ratio is increasing in a mean-preserving spread of F since $(p-c)^{-1}$ is convex in p and thus $\mathbb{E}_F[(p-c)^{-1}]$ increases but $\mathbb{E}_F[p-c]$ is constant. Equilibrium price dispersion, which is the dispersion of the distribution of transaction prices H, is thus driven by a specific measure of the dispersion of the distribution of posted prices F.

Example: Two-point distribution

Suppose that buyers are either non-captive with probability α (they meet two sellers) or captive with probability $1-\alpha$ (they meet one seller).

Describing this distribution in terms of the mean $\lambda = 1+\alpha$, we have

$$P_k(\lambda) = \begin{cases} 2-\lambda & k=1 \\ \lambda-1 & k=2 \end{cases}$$

To derive the equilibrium, Proposition 1 and $\lambda Q_k(\lambda) = kP_k(\lambda)$ yield

$$q(p) = \frac{\lambda - 2(\lambda-1)F(p)}{\lambda}.$$

Given that $Q_1(\lambda) = (2-\lambda)/\lambda$, we know that any posted price p satisfies

$$p = c + \frac{2-\lambda}{\lambda - 2(\lambda-1)F(p)}(v-c).$$

Rearranging the above, the distribution of posted prices is

$$F(p) = 1 - \frac{2-\lambda}{2(\lambda-1)}\left[\frac{v-c}{p-c} - 1\right].$$

Applying Lemma 1, the distribution of transaction prices is

$$H(p) = \lambda F(p) - (\lambda-1)F(p)^2.$$

By Proposition 2, the aggregate price can be expressed in terms of α as follows:

$$\tilde{p}(\lambda) = c + (1-\alpha)(v-c).$$

By Proposition 3, equilibrium price dispersion is given by

$$cv_H^2(\lambda) = \left(\frac{1}{2(\lambda-1)}\ln\left(\frac{\lambda}{2-\lambda}\right) - 1\right)\left(\frac{\tilde{p}(\lambda)-c}{\tilde{p}(\lambda)}\right)^2. \tag{15}$$

3.2 Invariant search technology

Consider the class of invariant search technologies introduced in Lester et al. (2015). This is a very wide class of search technologies which includes the Poisson distribution, the geometric distribution, and the negative binomial family.

Definition 1. *A search technology P_k is* invariant *if and only if for all $z \in [0,1]$,*

$$\sum_{k=0}^{\infty} P_k(\lambda) z^k = P_0(\lambda(1-z)) \tag{16}$$

where $\mathbb{E}_P[k] = \lambda$ and P_0 is continuous and infinitely differentiable for $\lambda > 0$.

We summarize some useful properties of invariant search technologies here.

Lemma 2. *If P_k is an invariant search technology, then (i) $P_0'(\lambda) < 0$ for all $\lambda > 0$; (ii) $P_0''(\lambda) > 0$ for all $\lambda > 0$; (iii) $\lim_{\lambda\to 0} P_0(\lambda) = 1$; (iv) $\lim_{\lambda\to\infty} P_0(\lambda) = 0$; (v) $\lim_{\lambda\to 0} P_0'(\lambda) = -1$, and (vi) $\lim_{\lambda\to\infty} P_0'(\lambda) = 0$. In terms of Q_k, we have (vii) $Q_1(\lambda) = -P_0'(\lambda)$ for all $\lambda > 0$ and therefore $\lim_{\lambda\to 0} Q_1(\lambda) = 1$ and $\lim_{\lambda\to\infty} Q_1(\lambda) = 0$.*

The invariance property is equivalent to the following condition in Lemma 3.

Lemma 3. *A search technology P_k is invariant if and only if for all $z \in [0,1]$,*

$$\sum_{k=1}^{\infty} Q_k(\lambda) z^{k-1} = Q_1(\lambda(1-z)) \tag{17}$$

where $\mathbb{E}_P[k] = \lambda$ and Q_1 is continuous and infinitely differentiable for $\lambda > 0$.

This form of the condition is more natural here because it involves the distribution $Q_k(\lambda)$ of the number of sellers a buyer meets from the sellers' perspective, which is the natural distribution for describing the equilibrium prices and quantities.

Invariance. The intuition behind the invariance condition can be described as follows. Suppose you are a seller and the expected number of sellers (including you) is λ. The distribution of the number of sellers k from your perspective (as a seller) is $Q_k(\lambda)$. Suppose that, for each of your competitors, the probability they are "worse" than you is z and the probability they are "better" is $1-z$.[5] What is the probability that you are the "best" seller? The answer is given by the left-hand side of (17), which is the probability that every one of your $k-1$ competitors is "worse".

Now consider a different problem. Suppose the search technology Q_k is the same but the expected number of sellers is now $\lambda(1-z)$. This is essentially saying that *only* the "better" sellers arrive. What is the probability that you are the "best" seller?

[5] In the present model, another seller is "better" if they post a lower price. However, this is just a particular example where $1-z = F(p)$. The invariance property is more general than this.

The answer is given by the right-hand side of (17), which is the probability you have *no* competitors arrive, i.e. the probability that $k = 1$ from a seller's perspective.

What does the invariance property say? Invariance says that the search technology has the property that the answer is the same for both problems. That is, the answer is "invariant" to whether you sort *after* applying the search technology (i.e. the first problem) or *before* applying the search technology (i.e. the second problem).

All of the equilibrium outcomes for the invariant class of search technologies can be expressed entirely in terms of the function Q_1, which determines the probability a seller has no competitors and is in a monopoly situation. For the Poisson search technology, for example, equilibrium outcomes can be expressed in terms of the function $Q_1(y) = e^{-y}$, which delivers the results for homogeneous sellers in Menzio (2024b).

Distribution of posted prices. Proposition 4 derives a simple and intuitive closed-form expression for the quantity demanded under an invariant search technology, which delivers a closed-form expression for the distribution of posted prices.

Proposition 4. *If P_k is invariant with mean λ, the equilibrium distribution of posted prices has a continuous cdf F with support $[\underline{p}, v]$ where $\underline{p} = c + Q_1(\lambda)(v - c)$.*

The distribution of posted prices F has the following closed-form expression:

$$F(p) = \frac{1}{\lambda}(Q_1)^{-1}\left(Q_1(\lambda)\frac{v-c}{p-c}\right). \tag{18}$$

Equivalently, any posted price p satisfies the equilibrium pricing condition:

$$p = c + \frac{Q_1(\lambda)}{q(p)}(v - c) \tag{19}$$

where $q(p)$ is the quantity demanded at posted price p:

$$q(p) = Q_1(\lambda F(p)) \tag{20}$$

and $Q_1(\lambda)$ is the probability a seller has no competitors, i.e. has a "captive" buyer.

Proof. From Proposition 1, the quantity demanded at posted price p is given by (4). Applying Lemma 3 with $z = 1 - F(p)$ delivers (20). Substituting into (19) gives

$$(p - c)Q_1(\lambda F(p)) = Q_1(\lambda)(v - c). \tag{21}$$

Because Q_1 is strictly decreasing by Lemma 2, it is invertible, and rearranging the above delivers the closed-form expression (18) for $F(p)$. $\square$

Quantiles. We can use Proposition 4 to state the equilibrium pricing condition in terms of quantiles. Define $p(x)$ as the unique solution to $x = F(p(x))$. That is, $p(x) \equiv F^{-1}(x)$, for any $x \in [0, 1]$. A seller at quantile $x = 0$ is the lowest-price seller, while a seller at quantile $x = 1$ is the highest-price seller. We can also define the corresponding quantity demanded at quantile x by $\hat{q}(x) \equiv q(p(x))$.

Corollary 1. *If P_k is invariant with mean λ, then posted prices satisfy*

$$p(x) = c + \frac{Q_1(\lambda)}{Q_1(\lambda x)} (v - c) \tag{22}$$

and the quantity demanded is given by

$$\hat{q}(x) = Q_1(\lambda x) \tag{23}$$

where x is the seller's quantile of the posted price distribution, i.e. $x = F(p(x))$.

Proof. Follows immediately from (19) and (20) in Proposition 4. $\square$

The invariance assumption greatly simplifies the equilibrium outcomes, which can be stated entirely in terms of the function Q_1. While $Q_1(\lambda)$ is the overall probability a seller has no competitors at all, $Q_1(\lambda x)$ is the probability that a seller has no competitors at quantile x or lower, i.e. no *lower-priced* or "better" competitors. The probability of having no lower-priced competitors is exactly the probability of sale (i.e. the quantity demanded) for a seller posting a price at quantile x.

Distribution of transaction prices. We can now derive the equilibrium distribution of transaction prices, which again takes on a simple form for this class.

Corollary 2. *If P_k is invariant with mean λ, the distribution of transaction prices has support $p \in [\underline{p}, v]$ and the cdf H of the distribution of transaction prices is*

$$H(p) = \frac{1 - P_0(\lambda F(p))}{1 - P_0(\lambda)}. \tag{24}$$

The density h of the distribution of transaction prices is

$$h(p) = \frac{Q_1(\lambda F(p))}{\tilde{q}(\lambda)} f(p). \tag{25}$$

Proof. From Lemma 1, the distribution of transaction prices is given by (5), where the numerator is $\sum_{k=1}^{\infty} P_k(\lambda)[1-(1-F(p))^k]$. Under invariance, Definition 1 implies $\sum_{k=0}^{\infty} P_k(\lambda)(1-F(p))^k = P_0(\lambda F(p))$, which yields the result. $\square$

For any invariant search technology, the aggregate price is given by the general Proposition 2, using $Q_1(\lambda)$ and $\tilde{q}(\lambda)$, and equilibrium price dispersion is given by Proposition 3, using the distribution of posted prices F described in Proposition 4.

3.3 Markups and markup dispersion

We focus on the class of invariant search technologies from now onwards. In order to align our results directly with the literature, such as Menzio (2024b), we now switch from prices and price dispersion to markups and markup dispersion.

We now assume that $c > 0$. Following the macroeconomics convention, we define the *markup* by $\mu \equiv p/c$. All of the results in this section follow from Section 3.2.

Markups. In the Burdett-Judd model, markups can take any value between the competitive markup, $p/c = 1$, and the monopoly markup, which we denote by $\bar{\mu} \equiv v/c$. The *aggregate markup* $\tilde{\mu}(\lambda) = \mathbb{E}_H(\mu)$ is a quantity-weighted average of the firm-level markups μ posted by sellers. Expression (6) implies that

$$\tilde{\mu}(\lambda) = \int \frac{q(p)}{\tilde{q}(\lambda)} \left(\frac{p}{c}\right) f(p)dp = \frac{1}{c} \int ph(p)dp = \frac{\tilde{p}(\lambda)}{c}. \tag{26}$$

Therefore, $\tilde{\mu}(\lambda) = \tilde{p}(\lambda)/c$ is the aggregate markup, which is quantity-weighted.

Corollary 3. *If P_k is invariant with mean λ, the aggregate markup is*

$$\tilde{\mu}(\lambda) = 1 + (1-\alpha(\lambda))\,(\bar{\mu}-1) \tag{27}$$

where $1-\alpha(\lambda) = Q_1(\lambda)/\tilde{q}(\lambda)$ is the proportion of monopoly sales.

Proof. Follows from Proposition 2 using $\tilde{\mu}(\lambda) = \tilde{p}(\lambda)/c$ and $\bar{\mu} \equiv v/c$. $\square$

Corollary 1 yields the following result regarding markups by quantile. The invariance assumption greatly simplifies the quantile markup function $\mu(x)$.

Corollary 4. *If P_k is invariant with mean λ, the equilibrium markup is*

$$\mu(x) = 1 + \frac{Q_1(\lambda)}{Q_1(\lambda x)}\,(\bar{\mu}-1)\,. \tag{28}$$

Proof. Follows directly from Corollary 1 using the fact that $\mu(x) = p(x)/c$. $\square$

Markup dispersion. Since the markup is defined as $\mu \equiv p/c$, the transaction markup distribution is just a rescaling of the transaction price distribution. Recall that H is the cdf of the distribution of transaction prices and let H_μ denote the cdf of the distribution of transaction markups. We have the following relationship:

$$H_\mu(p/c) \;\equiv\; \Pr\left(\mu \leq p/c\right) = \Pr\left(\mu c \leq p\right) = H(p). \tag{29}$$

We can obtain the moments of the distribution of transaction markups H_μ directly from the distribution of transaction prices H. For any $k \in \mathbb{N}$, we have

$$\mathbb{E}_{H_\mu}[\mu^k] = \frac{1}{c^k}\mathbb{E}_H[p^k]. \tag{30}$$

Therefore, we have

$$cv^2_{H_\mu}(\lambda) = \frac{\mathbb{E}_{H_\mu}[\mu^2]}{\tilde{\mu}(\lambda)^2} - 1 = \frac{c^2\mathbb{E}_H[p^2]}{c^2\tilde{p}(\lambda)^2} - 1 = cv^2_H(\lambda). \tag{31}$$

Given that price dispersion and markup dispersion are the same using our preferred measure, we now let $cv_H(\lambda)$ denote both price dispersion and markup dispersion. Both are given by the same expression, which is derived in Proposition 3.

Example: Poisson search technology

For the Poisson search technology with mean $\lambda > 0$, we have

$$P_k(\lambda) = \frac{\lambda^k e^{-\lambda}}{k!}.$$

It is invariant by Definition 1 because its probability generating function is

$$\mathbb{G}(z) = \sum_{k=0}^{\infty} P_k(\lambda) z^k = \sum_{k=0}^{\infty} \frac{\lambda^k e^{-\lambda}}{k!} z^k = e^{-\lambda(1-z)} = P_0(\lambda(1-z)).$$

Next, $Q_1(\lambda) = P_1(\lambda)/\lambda = e^{-\lambda}$ and $q(p) = Q_1(\lambda F(p))$ from Proposition 4 implies

$$q(p) = e^{-\lambda F(p)}.$$

Given that $Q_1(\lambda) = e^{-\lambda}$, any posted price p satisfies

$$p = c + e^{-\lambda(1-F(p))}(v - c).$$

Rearranging the above, the distribution of posted prices is

$$F(p) = 1 - \frac{1}{\lambda} \ln\left(\frac{v-c}{p-c}\right).$$

Applying Corollary 4 and using the fact that $Q_1(\lambda) = e^{-\lambda}$, we have

$$\mu(x) = 1 + e^{-\lambda(1-x)} \left(\bar{\mu} - 1\right). \tag{32}$$

Applying Corollary 2, the distribution of transaction prices is

$$H(p) = \frac{1 - e^{-\lambda F(p)}}{1 - e^{-\lambda}}. \tag{33}$$

By Proposition 2, the aggregate price is

$$\tilde{p}(\lambda) = c + (1 - \alpha(\lambda))(v - c) \tag{34}$$

where

$$1 - \alpha(\lambda) = \frac{\lambda e^{-\lambda}}{1 - e^{-\lambda}}. \tag{35}$$

By Proposition 3, equilibrium price dispersion is given by

$$cv_H^2(\lambda) = \left(\frac{(1 - e^{-\lambda})^2}{\lambda^2 e^{-\lambda}} - 1\right)\left(\frac{\tilde{p}(\lambda) - c}{\tilde{p}(\lambda)}\right)^2. \tag{36}$$

Observe that (32) is the markup expression for homogeneous firms in Menzio (2024b), which assumes the search technology is Poisson. In Appendix B, we also consider the geometric search technology and the negative binomial family.

4 Effect of competition

Before we turn to our analysis of the effects of competitive dispersion, we first consider the effect of changes in the *degree of competition* λ on equilibrium outcomes such as the aggregate markup, seller markups, and price dispersion.

4.1 Aggregate markup

For the Poisson, geometric, and two-point examples, the aggregate markup is strictly decreasing in the degree of competition λ. Surprisingly, for a general search technology, the aggregate markup may not be strictly decreasing for any degree of competition. Greater competition – in the sense of a higher average number of firms each buyer meets – may not necessarily decrease the aggregate markup. In fact, even within the class of invariant search technologies, the aggregate markup may or may not be decreasing in the degree of competition without imposing a further condition.

Before presenting our next result, we define the *matching elasticity*:

$$\eta_m(\lambda) \equiv \frac{\lambda m'(\lambda)}{m(\lambda)} = \frac{-\lambda P_0'(\lambda)}{1 - P_0(\lambda)}. \tag{37}$$

We also present the following lemma regarding the proportion of monopoly sales. For an invariant search technology, this turns out to be equal to the matching elasticity.

Lemma 4. *If P_k is invariant with mean λ, the proportion of monopoly sales is*

$$1 - \alpha(\lambda) = \eta_m(\lambda) \tag{38}$$

and therefore the aggregate markup is

$$\tilde{\mu}(\lambda) = 1 + \eta_m(\lambda)\left(\bar{\mu} - 1\right) \tag{39}$$

where $\eta_m(\lambda)$ is the matching elasticity.

Proof. By definition, we have $1 - \alpha(\lambda) = Q_1(\lambda)/\tilde{q}(\lambda)$. Also, we have $\tilde{q}(\lambda) = m(\lambda)/\lambda$ where $m(\lambda) = 1 - P_0(\lambda)$. If P_k is an invariant search technology, then $Q_1(\lambda) = -P_0'(\lambda)$ by Lemma 2. Therefore, we have the following:

$$1 - \alpha(\lambda) = \frac{Q_1(\lambda)}{\tilde{q}(\lambda)} = \frac{-\lambda P_0'(\lambda)}{1 - P_0(\lambda)}. \tag{40}$$

Applying definition (37) for the matching elasticity, we obtain the result (38). Expression (39) then follows immediately from Corollary 3. □

We are now in a position to state the following result.

Lemma 5. *If P_k is invariant with mean λ, greater competition decreases the aggregate markup if and only if the matching elasticity η_m is strictly decreasing in λ.*

Proof. If P_k is invariant, it is clear from expression (39) that the aggregate markup $\tilde{\mu}(\lambda)$ is strictly decreasing in λ if and only if η_m is strictly decreasing in λ. □

4.2 Seller markups

To derive clear monotonic comparative static results with respect to λ regarding quantile markups and the distributions of posted prices and transaction prices, we need to make an additional assumption. Applying Lemma 2, the elasticity of Q_1 is

$$\eta_{Q_1}(\lambda) \equiv \frac{\lambda Q_1'(\lambda)}{Q_1(\lambda)} = \frac{\lambda P_0''(\lambda)}{P_0'(\lambda)}. \tag{41}$$

The elasticity η_{Q_1} of the function Q_1 is equal to the elasticity $\eta_{m'}$ of the function m'.

Assumption 1. *The elasticity η_{Q_1} is strictly decreasing for all $\lambda > 0$.*

This assumption does not hold for all invariant search technologies, but it holds for our main examples: the Poisson distribution, the geometric distribution, and the negative binomial family, which we consider in Section 5.

For the next result, we use notation that emphasizes the dependence of F and H on λ, which we usually suppress for simplicity. To signify that a distribution F_1 first-order stochastic dominates F_2, we write $F_2 \preccurlyeq_{FOSD} F_1$ (i.e. F_2 has lower prices).

Proposition 5. *If Assumption 1 holds and P_k is invariant with mean λ, then:*

1. *The aggregate markup is strictly decreasing in λ.*
2. *For any $x \in (0,1)$, the quantile markup is strictly decreasing in λ.*
3. *For any $\lambda_2 > \lambda_1$, the distribution of posted prices $F(\cdot;\lambda_2) \preccurlyeq_{FOSD} F(\cdot;\lambda_1)$.*
4. *For any $\lambda_2 > \lambda_1$, the distribution of transaction prices $H(\cdot;\lambda_2) \preccurlyeq_{FOSD} H(\cdot;\lambda_1)$.*

Figure 1 illustrates how the aggregate markup varies with the degree of competition for both the Poisson and the geometric search technologies. For the geometric, the aggregate markup falls more sharply compared to the Poisson at lower levels of competition, but at higher degrees of competition the aggregate markup falls to $p/c = 1$ rapidly for the Poisson. For the geometric, the aggregate markup stays elevated above the competitive markup even at relatively high levels of competition, although it also converges to one in the competitive limit as $\lambda \to \infty$.

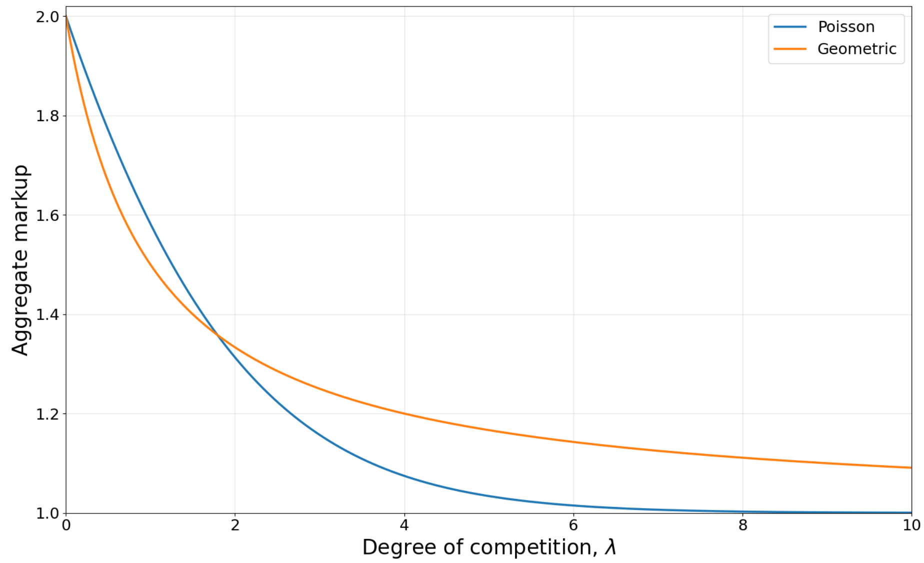

Figure 1: Aggregate markup and competition, Poisson vs geometric ($c = 1$ and $v = 2$)

4.3 Monopoly and competitive limits

We now consider two limiting cases regarding the degree of competition: the monopoly limit and the competitive limit.

Monopoly limit. First, consider the monopoly limit, i.e. the limit as the degree of competition $\lambda \to 0$. In this limiting case, all buyers are "captive" and meet only one seller. For both the standard examples, the two-point distribution and the Poisson, the distribution of posted prices F converges to a degenerate distribution at the monopoly price v, the quantile markup converges to the monopoly markup $\bar{\mu} = v/c$ for any quantile $x \in (0, 1)$, and the aggregate markup converges to the monopoly markup. Proposition 6 says these results are true for any invariant technology.

Proposition 6. *Suppose that P_k is invariant with mean λ. In the monopoly limit as $\lambda \to 0$, the following hold:*

1. *The distribution of posted prices F is degenerate at $p = v$.*

2. *For any $x \in (0, 1)$, the quantile markup converges to the monopoly markup.*

3. *The distribution of transaction prices H is degenerate at $p = v$.*

4. *The aggregate markup converges to the monopoly markup, $\bar{\mu} = v/c$.*

Competitive limit. Now consider the competitive limit, i.e. the limit as the degree of competition $\lambda \to \infty$. In this limiting case, all buyers meet two or more sellers. For both standard examples, the two-point distribution and the Poisson, the distribution of posted prices F converges to a degenerate distribution at the competitive price c and the quantile markup converges to the competitive markup $p/c = 1$ for any quantile $x \in (0, 1)$. Surprisingly, these results are *not* necessarily true for any invariant technology. In particular, they are not true for the geometric search technology and the negative binomial search technology more generally. While the aggregate markup converges to the competitive markup for any invariant search technology, and the distribution of transaction prices converges to a degenerate distribution, the distribution of *posted prices* need not converge to a degenerate distribution.

Before we present our results regarding the competitive limit, we first define

$$\psi(x) \equiv \lim_{t\to\infty} \frac{Q_1(t)}{Q_1(tx)} \tag{42}$$

for all $x \in (0, 1)$, provided this limit exists. Lemma 6 says Assumption 1 is sufficient.

Lemma 6. *If Assumption 1 holds, then the limit $\psi(x)$ exists for all $x \in (0, 1)$. Moreover, either $\psi(x) = 0$ for all $x \in (0, 1)$ or ψ is strictly increasing on $(0, 1)$.*

We can now characterize the competitive limit under Assumption 1. By Lemma 6, the following result covers all possible cases that apply under Assumption 1.

Proposition 7. *Suppose that Assumption 1 holds and P_k is invariant with mean λ. In the competitive limit as $\lambda \to \infty$, the following hold:*

1. *If $\psi(x) = 0$ for all $x \in (0, 1)$, the distribution of posted prices F is degenerate at $p = c$.*

2. *If ψ is strictly increasing on $(0, 1)$, the distribution of posted prices F converges to a non-degenerate distribution F_∞ with support $[c, v]$ and*

$$F_\infty(p) = \psi^{-1}\left(\frac{p-c}{v-c}\right). \tag{43}$$

3. *For any $x \in (0, 1)$, the quantile markup converges to*

$$\mu_\infty(x) = 1 + \psi(x)\left(\bar{\mu} - 1\right). \tag{44}$$

4. *The distribution of transaction prices H is degenerate at $p = c$.*

5. *The aggregate markup converges to the competitive markup, $p/c = 1$.*

In the competitive limit as $\lambda \to \infty$, the equilibrium may converge to one in which sellers choose a *mixed strategy* given by the non-degenerate distribution of posted prices F_∞, rather than the pure strategy $p = c$, which emerges as the limit of the mixed-strategy equilibrium as $\lambda \to \infty$ for the Poisson case. In such cases where F_∞ is non-degenerate, sellers are indifferent between all $p \in [c, v]$.

At the same time, however, the quantity demanded $q(p)$ for each posted price (except c) converges to zero. The possibility of a non-degenerate distribution of posted prices F is therefore consistent with a degenerate distribution of transaction prices H and an aggregate markup equal to the competitive markup, as for the Poisson case. In the competitive limit, sellers may post a range of prices but no one buys from any sellers who post $p > c$. All sellers make zero profits because $\pi(p) = (p - c)q(p) \to 0$ either way (either because $p \to c$ *or* $q(p) \to 0$) and therefore sellers are indifferent.

For the Poisson search technology, $\psi(x) = 0$ because $Q_1(t) = e^{-t}$. Therefore, the quantile markup $\mu(x)$ converges to $p(x)/c = 1$ for all $x \in (0, 1)$ and the distribution of posted prices converges to a degenerate distribution at $p = c$. In general, however, this is not necessarily true for any search technology, even within the invariant class. For example, for the geometric search technology, the quantile markup is in Appendix B. In the limit as $\lambda \to \infty$, we have $\mu(x) \to 1 + x^2(\bar{\mu} - 1)$ or $p(x) \to c + x^2(v - c)$.

Figure 2 illustrates how the limiting quantile markup $\mu_\infty(x)$ for $x > 0$ need not equal the competitive markup ($p/c = 1$) if the search technology P_k is not Poisson.

4.4 Price dispersion

Consider the effect of competition on price dispersion. We recover an intuitive feature of the Burdett-Judd model: price dispersion disappears in both the monopoly limit as $\lambda \to 0$ and the competitive limit as $\lambda \to \infty$. Proposition 8 says that there is no price dispersion in either of these limits for any invariant search technology. While there may be dispersion in *posted prices* in the competitive limit, as Proposition 7 shows, there is no dispersion in transaction prices because H is degenerate.

Proposition 8. *Suppose that Assumption 1 holds and P_k is invariant with mean λ.*

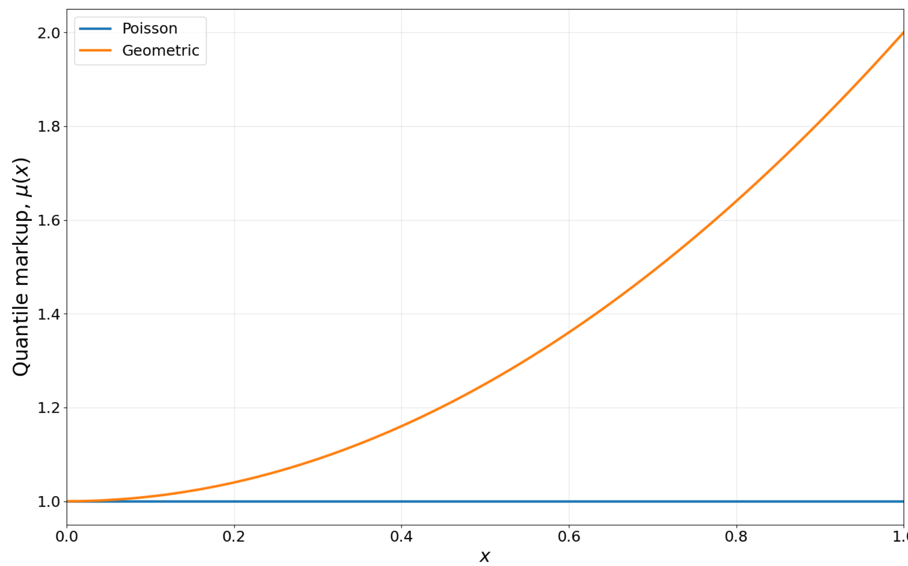

Figure 2: Quantile markup function as $\lambda \to \infty$, Poisson vs geometric ($c = 1$ and $v = 2$)

There is no price dispersion in either the competitive limit or the monopoly limit:

$$\lim_{\lambda \to 0} cv_H^2(\lambda) = 0 \qquad \textit{and} \qquad \lim_{\lambda \to \infty} cv_H^2(\lambda) = 0.$$

Therefore price dispersion varies non-monotonically with the degree of competition λ.

Proof. By Proposition 6, as $\lambda \to 0$ the transaction price distribution H is degenerate at v, hence $cv_H^2(\lambda) \to 0$ as $\lambda \to 0$. Next, by Proposition 7, the distribution H is degenerate at c in the limit as $\lambda \to \infty$, and therefore $cv_H^2(\lambda) \to 0$ as $\lambda \to \infty$. $\square$

Proposition 8 guarantees that price dispersion goes to zero in both of these limits, which implies that price dispersion varies non-monotonically with the degree of competition. That is, there exist $\lambda > 0$ where greater competition increases price dispersion and there exist $\lambda < \infty$ where greater competition decreases price dispersion.

Proposition 8 also implies there exists *at least one* interior global maximizer of price dispersion. It is difficult to provide general conditions under which a *unique* global maximizer of price dispersion exists, i.e. a degree of competition which maximizes price dispersion. However, this is true for our three main examples: the two-point distribution, the Poisson, and the geometric search technology.

Figure 3 illustrates how price dispersion varies with the degree of competition for both the Poisson and the geometric search technologies. Price dispersion goes to zero both as $\lambda \to 0$ and as $\lambda \to \infty$, consistent with Proposition 8. For the geometric,

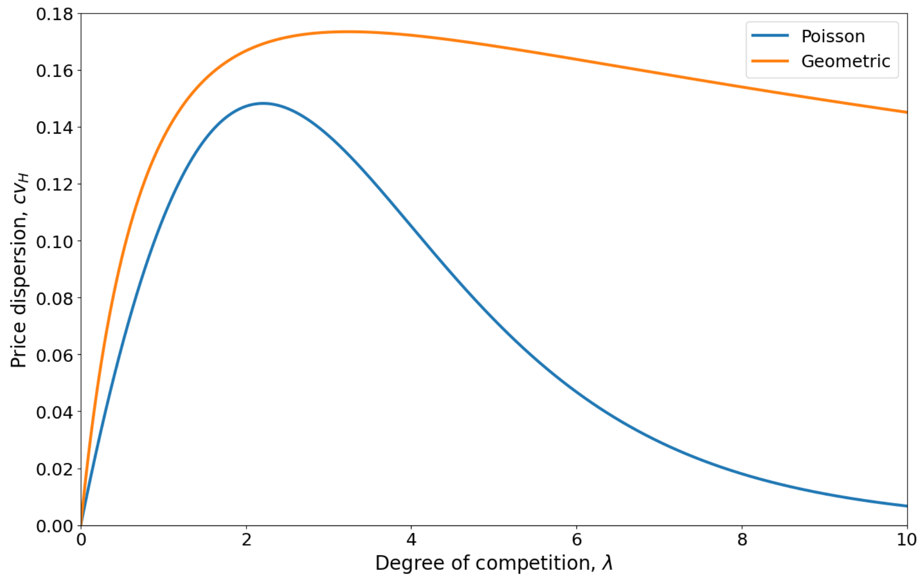

Figure 3: Price dispersion and competition, Poisson vs geometric ($c = 1$ and $v = 2$)

the decline in price dispersion as λ becomes large is much slower, so price dispersion stays high even when the market is very competitive, unlike for the Poisson.

5 Effect of competitive dispersion

We can now provide some answers to our motivating questions about the effects of competitive dispersion. How does competitive dispersion affect markups and price dispersion? Does greater competitive dispersion raise or lower the aggregate markup? Does more competitive dispersion always lead to more price dispersion?

To answer these questions, we need to move beyond the standard distributions (the two-point distribution and the Poisson) so that we can separate the effects of the *degree of competition* from the effects of *competitive dispersion*. We take λ, the degree of competition, to be exogenous. (In Section 6, we consider seller entry.)

Our measure of competitive dispersion is the coefficient of variation $cv(\lambda)$ of the distribution $P_k(\lambda)$. For any invariant search technology, this can be calculated using the following lemma, which tells us that a mean-preserving change in the degree of competitive dispersion is equivalent to a change in the term $Q_1'(0) \equiv \lim_{\lambda \to 0} Q_1'(\lambda)$.

Lemma 7. *If P_k is invariant with mean λ, competitive dispersion $cv(\lambda)$ is given by*

$$cv^2(\lambda) = \frac{1}{\lambda} - Q_1'(0) - 1. \tag{45}$$

The effect of competitive dispersion on both markups and price dispersion is complex and depends on the particular search technology. The aggregate markup may be either increasing or decreasing in competitive dispersion, depending on the search technology and the degree of competition. At the same time, an increase in competitive dispersion does *not* always lead to an increase in price dispersion.

5.1 Aggregate markup

To study the effects of competitive dispersion, we consider two different exercises. The first considers an example of a "large" increase in dispersion: a shift from the Poisson to the geometric search technology, which has the same mean λ but higher dispersion. The second considers the general comparative static effect of a change in competitive dispersion in a two-parameter family, the negative binomial family.

Poisson versus geometric. First, we compare the aggregate markup for the Poisson search technology and the aggregate markup for the geometric search technology (which has higher competitive dispersion but the same mean λ). Applying Lemma 7, competitive dispersion is given by $cv_G^2(\lambda) = 1/\lambda + 1$ for the geometric compared to $cv_P^2(\lambda) = 1/\lambda$ for the Poisson. See Appendix B for the geometric example.

We find that a "large" increase in competitive dispersion (i.e. a shift from the Poisson to the geometric) may either increase or decrease the aggregate markup, depending on the degree of competition. Proposition 9 shows that there exists a unique cut-off degree of competition at which the direction of the effect changes.

Proposition 9. *Suppose there is an increase in competitive dispersion represented by a shift from the Poisson to the geometric search technology with the same degree of competition λ. Let $\lambda_c^* \approx 1.79$ be the unique solution to $e^\lambda = 1 + \lambda + \lambda^2$.*

1. *If $\lambda < \lambda_c^*$, the aggregate markup is strictly lower.*
2. *If $\lambda > \lambda_c^*$, the aggregate markup is strictly higher.*

This result tells us that in less competitive markets (lower λ) greater competitive dispersion *decreases* the aggregate markup, but in more competitive markets (higher λ) greater competitive dispersion *increases* the aggregate markup. This result, however, captures only the effect of a specific "large" jump in competitive dispersion (namely, a shift from the Poisson to the geometric). Figure 4 illustrates this result.

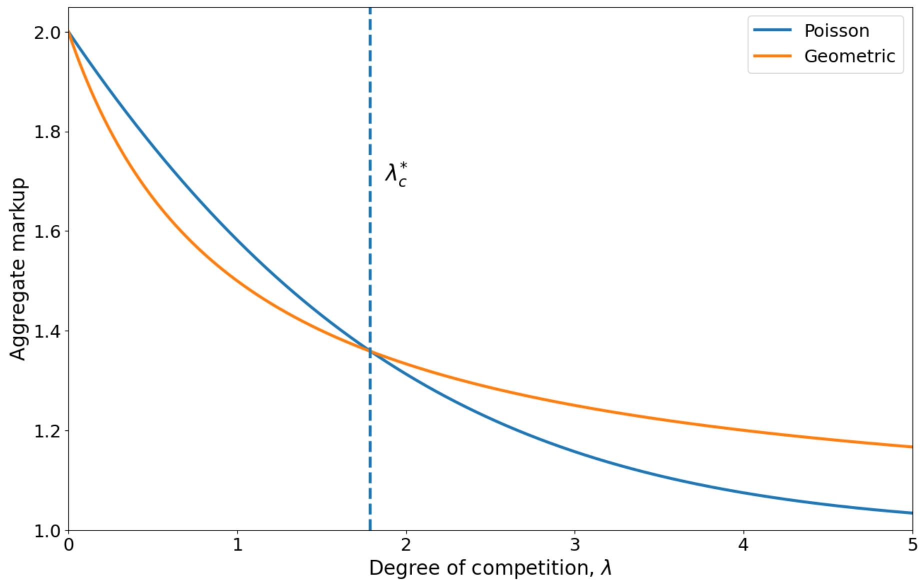

Figure 4: Aggregate markup and competition, Poisson vs geometric ($c = 1$ and $v = 2$)

To understand this result, consider the fact that the effect of competitive dispersion on the aggregate markup depends on its effect on the following term:

$$1 - \alpha(\lambda) = \frac{Q_1(\lambda)}{\tilde{q}(\lambda)} = \left[\frac{Q_1(\lambda)}{m(\lambda)}\right]\lambda. \tag{46}$$

The fraction in square brackets is the only part that moves with dispersion. The denominator $m(\lambda)$ is decreasing in competitive dispersion in the sense that a shift from the Poisson to the geometric leaves more buyers unmatched, i.e. $m(\lambda)$ is lower. However, the numerator $Q_1(\lambda)$, i.e. the probability a seller is in a local monopoly, may either increase or decrease with greater competitive dispersion.

When the average number of sellers in a meeting is low (i.e. λ is close to one), the probability a seller is in a monopoly meeting is already high and an increase in dispersion tends to move mass away from $k = 1$, thus decreasing $Q_1(\lambda)$. On the other hand, when the average number of sellers is high (i.e. λ is far above one), the probability a seller is in a monopoly meeting is low and greater dispersion tends to move mass towards $k = 1$, thus increasing $Q_1(\lambda)$. The overall effect on the aggregate markup depends on the relative size of the effects on $Q_1(\lambda)$ and $m(\lambda)$.

Negative binomial family. To determine the general comparative static effect of a change in competitive dispersion, we need to consider a two-parameter family of distributions. This will enable us to separate the mean and the dispersion of the distribution $P_k(\lambda)$ and allow us to vary the dispersion continuously. We consider the negative binomial family, which satisfies Assumption 1 and therefore all of the propositions in Section 4 regarding the effects of competition hold for this family.

For the negative binomial family of distributions $P_k(\lambda; r)$ with mean λ and parameter $r \in (0, \infty)$, the probability a buyer meets $k \in \mathbb{N}$ sellers is given by[6]

$$P_k(\lambda; r) = \binom{k+r-1}{k} \left(\frac{r}{r+\lambda}\right)^r \left(\frac{\lambda}{r+\lambda}\right)^k$$

and

$$P_0(\lambda; r) = \left(\frac{r}{r+\lambda}\right)^r.$$

The expressions for all of the equilibrium outcomes can be found in Appendix B. In the special case where $r = 1$, we recover the geometric search technology. In the limit as $r \to \infty$, we recover the Poisson search technology.

Since $-Q_1'(0) = P_0''(0) = 1 + 1/r$, competitive dispersion $cv(\lambda)$ is given by

$$cv^2(\lambda; r) = \frac{1}{\lambda} + \frac{1}{r}.$$

For this family of search technologies, $1/r \in (0, \infty)$ determines the competitive dispersion, i.e. the dispersion in the number of sellers a buyer meets. To vary competitive dispersion while holding the degree of competition λ fixed, we can just vary $1/r$.

What is the effect of competitive dispersion on the aggregate markup? We find that the effect depends on the degree of competition λ *and* the degree of competitive dispersion via $1/r$. The next proposition, Proposition 10, describes our result.

Proposition 10. *Suppose that P_k is negative binomial with mean λ and competitive dispersion $1/r$. Let $\lambda_d^* \approx 1.59$ be the unique solution to $e^\lambda = 2/(2-\lambda)$.*

1. *If $\lambda < \lambda_d^*$, the aggregate markup is strictly decreasing in competitive dispersion.*
2. *If $\lambda > \lambda_d^*$, there exists a unique $r^*(\lambda) \in (0, \infty)$ such that the aggregate markup is increasing in dispersion on $(0, 1/r^*)$ and decreasing in dispersion on $(1/r^*, \infty)$.*

---

[6]For real $r > 0$, we interpret the binomial coefficient in $P_k(\lambda; r)$ as the generalized binomial coefficient, which coincides with the usual combinatorial definition when $r \in \mathbb{N}$:

$$\binom{k+r-1}{k} \equiv \frac{\Gamma(k+r)}{\Gamma(r)\Gamma(k+1)}$$

where Γ denotes the Gamma function and $\Gamma(n) = (n-1)!$ for positive integers $n \geq 1$.

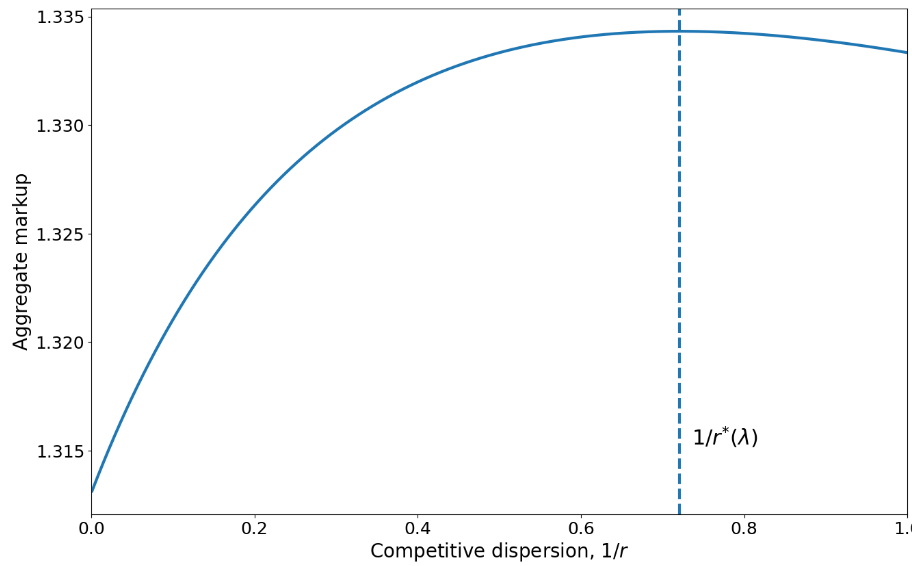

Figure 5: Aggregate markup and competitive dispersion, NB family ($\lambda = 2$, $c = 1$, $v = 2$)

Proposition 10 tells us that in less competitive markets (lower λ), greater competitive dispersion always decreases the aggregate markup. On the other hand, in more competitive markets (higher λ), the effect of competitive dispersion on the aggregate markup is non-monotonic. For lower levels of competitive dispersion, i.e. lower levels of $1/r$, the aggregate markup is increasing in competitive dispersion, but for higher levels of competitive dispersion it is decreasing. Figure 5 provides an example.

5.2 Price dispersion

Competitive dispersion is necessary for price dispersion in the Burdett-Judd model. Given this, we might expect that more competitive dispersion would lead to more price dispersion. Surprisingly, however, we find that this is not necessarily the case.

Poisson versus geometric. First, we consider the effect of a "large" increase in competitive dispersion (i.e. a shift from the Poisson to the geometric) on equilibrium price dispersion. Proposition 11 shows that, in some cases, the Poisson search technology generates higher price dispersion than the geometric.

Proposition 11. *Suppose there is an increase in competitive dispersion represented by a shift from the Poisson to the geometric search technology with the same mean λ.*

1. *If $\lambda \leq 2$, then equilibrium price dispersion increases.*

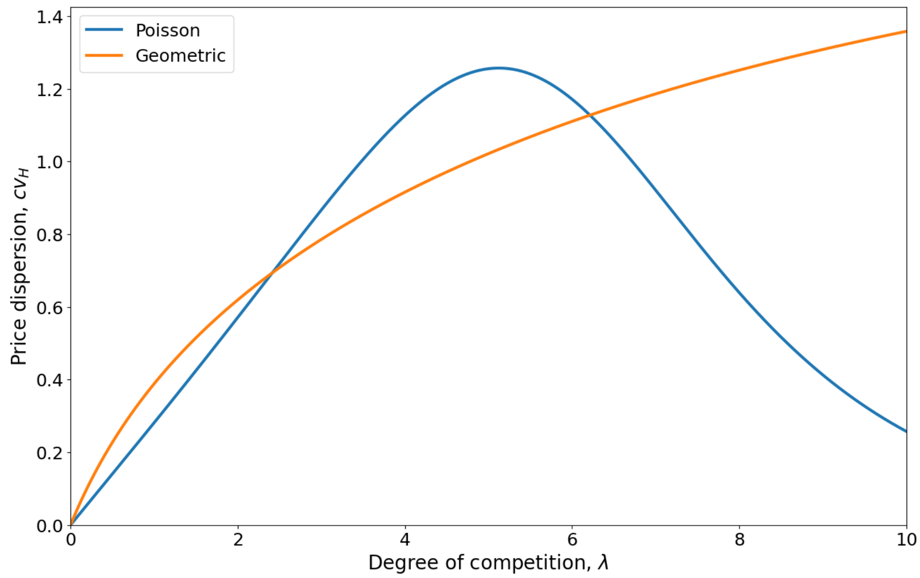

Figure 6: Price dispersion and competition, Poisson vs geometric ($c = 0.05$ and $v = 2$)

2. *For any $\lambda > 0$, equilibrium price dispersion increases if and only if*

$$\frac{1 + (\bar{\mu} - 1)\,\hat{\eta}_m(\lambda)}{\bar{\mu} + \lambda} > \frac{\sqrt{3\,(1+\lambda)\,(e^{-\lambda} - \;\hat{\eta}_m(\lambda)^2)}}{\lambda}$$

where $\hat{\eta}_m(\lambda) \equiv \frac{\lambda e^{-\lambda}}{1-e^{-\lambda}}$, and decreases if and only if the reverse holds.

Figure 6 provides an illustration of this result. When the degree of competition $\lambda \leq 2$, the geometric search technology delivers higher equilibrium price dispersion than the Poisson. For higher values of λ, price dispersion can sometimes be higher for the Poisson than for the geometric, which is surprising given that the Poisson has significantly lower competitive dispersion for any degree of competition λ.

Negative binomial family. For the negative binomial family, it is difficult to get general comparative statics results regarding the effect of competitive dispersion $1/r$ on price dispersion. However, our next result describes the *local effect* of competitive dispersion on equilibrium price dispersion around the Poisson benchmark as $c \to 0$. This is sufficient to show that price dispersion can either increase or decrease locally with greater competitive dispersion, depending on the degree of competition.

Proposition 12. *Suppose that P_k is negative binomial with mean λ and competitive dispersion $1/r$. Consider the limit as $c \to 0$. In a neighborhood of $1/r = 0$ (Poisson),*

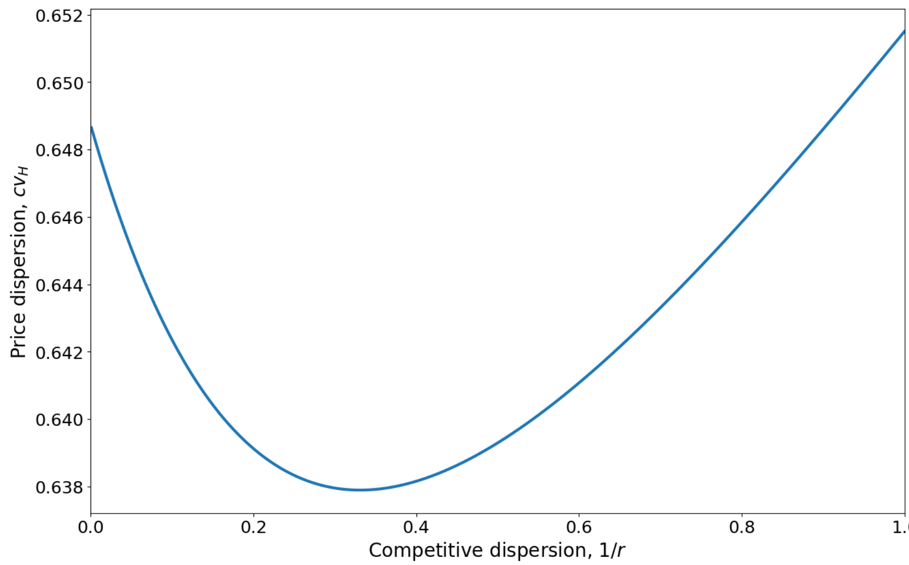

Figure 7: Price dispersion and competitive dispersion, NB family ($\lambda = 2.5$, $c = 0.05$, $v = 1$)

1. *If $\lambda < 2$, price dispersion is strictly increasing in competitive dispersion.*

2. *If $\lambda > 2$, price dispersion is strictly decreasing in competitive dispersion.*

Figure 7 shows an example where c is low, which illustrates Proposition 12. For this example, $\lambda = 2.5$ and price dispersion is decreasing in competitive dispersion locally near the Poisson. By contrast, it increases locally near the geometric ($1/r = 1$).

Given that the search technology P_k allows some buyers to meet zero sellers, one might wonder whether changes in competitive dispersion via the parameter $1/r$ primarily affect the probability $P_0(\lambda; r)$, so that competitive dispersion among *trading buyers* may not necessarily increase with the dispersion parameter $1/r$. However, for the negative binomial family, increasing $1/r$ not only increases unmatched buyers via $P_0(\lambda; r)$ but also increases dispersion in meeting size *conditional on matching*. The fact that greater competitive dispersion may decrease price dispersion is even more surprising given that competitive dispersion increases among buyers who purchase.[7]

[7]Let $\bar{P}_k(\lambda; r) \equiv P_k(\lambda; r \mid K \geq 1)$ denote the zero-truncated distribution of the number of price quotes and let $\bar{K}$ denote a draw from this distribution. For fixed λ, the coefficient of variation of this zero-truncated distribution, $cv_{\bar{K}}(\lambda; r)$, is increasing in the dispersion parameter $1/r$.

6 Entry and efficiency

Up to this point, we have taken λ, the average number of sellers a buyer meets, to be exogenous. We now consider whether the level of seller entry would be constrained efficient, taking the search technology P_k and the measure of buyers as given, if there was a zero-profit condition determining seller entry. That is, would the equilibrium level of seller entry decentralize the social planner's solution?

For the Poisson search technology, Menzio (2024b) shows that seller entry is constrained efficient. However, constrained efficiency of seller entry is not guaranteed in general. Instead, it requires a certain condition on the search technology. This follows from a more general result in Lester et al. (2015). We can see this clearly in the Burdett-Judd model and relate it to the well-known Hosios (1990) condition.

6.1 Equilibrium seller entry

First, we consider the equilibrium level of seller entry. Suppose the cost of entry for a seller is $\kappa > 0$ and assume that $v - c > \kappa$. The zero profit condition is

$$\tilde{q}(\lambda)(\tilde{p}(\lambda) - c) = \kappa. \tag{47}$$

Given that all sellers make equal expected profit in equilibrium, we also have

$$Q_1(\lambda)(v - c) = \kappa. \tag{48}$$

If Q_1 is strictly decreasing, the equilibrium level of seller entry $\lambda^* = s^*/b$ is

$$\lambda^* = Q_1^{-1}\left(\frac{\kappa}{v - c}\right). \tag{49}$$

The equilibrium degree of competition λ^* is strictly decreasing in the cost of entry κ and the marginal cost c, and strictly increasing in buyers' valuation v. The degree of competition λ^* may be either increasing or decreasing in competitive dispersion, depending on the search technology and the parameters, even within the negative binomial family. Greater competitive dispersion may attract more or less seller entry.

6.2 Social planner's problem

Define the total social surplus *per buyer* as follows:

$$W(\lambda) = m(\lambda)(v - c) - \kappa\,\lambda.$$

Taking the search technology P_k as given, the planner chooses λ to maximize $W(\lambda)$. The first-order condition for the planner's solution is:

$$m'(\lambda)(v - c) = \kappa. \tag{50}$$

To ensure there is a unique solution to the first-order condition (50), suppose that $m'(\lambda) > 0$ and $m''(\lambda) < 0$ for all $\lambda > 0$, with $\lim_{\lambda\to 0} m'(\lambda) = 1$ and $\lim_{\lambda\to\infty} m'(\lambda) = 0$. Then $W(\lambda)$ is strictly concave and there is a unique interior planner's solution.

Now consider the relationship between the zero profit condition and the planner's first-order condition. Equating (50) and (47) and rearranging, and using the fact that $\tilde{q}(\lambda) = m(\lambda)/\lambda$, the equilibrium decentralizes the planner's solution if and only if

$$\frac{\tilde{p}(\lambda) - c}{v - c} = \frac{m'(\lambda)\lambda}{m(\lambda)}. \tag{51}$$

This is the familiar Hosios (1990) condition, which says that we have constrained efficiency if and only if the equilibrium surplus share of sellers (the left-hand side of the above equation) is equal to the matching elasticity (the right-hand side).[8]

For a general search technology, expression (8) for $\tilde{p}(\lambda)$ in Proposition 2 implies that the equilibrium surplus share of sellers is equal to $1 - \alpha(\lambda)$. Therefore, we have constrained efficiency if and only if the following condition holds.

Proposition 13. *Seller entry is constrained efficient if and only if, for all $\lambda > 0$, the proportion of monopoly sales is equal to the matching elasticity:*

$$1 - \alpha(\lambda) = \eta_m(\lambda). \tag{52}$$

The left-hand-side is the equilibrium surplus share of firms according to Proposition 2, which is equal to the probability that a sale is a monopoly sale. The right-hand-side is the matching elasticity. The left term governs firm profits while the right term governs how the social surplus increases with firm entry by increasing the total number of sales. Only when these two are aligned do we get efficiency of entry.

[8] Note that under Nash bargaining the left-hand side would be equal to sellers' bargaining power.

For the Poisson search technology, the left-hand-side and right-hand-side of (52) are identical. However, this is not always true for any search technology. In general, there is no reason why these two things – the probability of monopoly and the matching elasticity – should be equal to each other in the Burdett-Judd model.

For the invariant class of search technologies, this efficiency condition always holds.

Corollary 5. *If P_k is invariant, then seller entry is constrained efficient.*

Proof. Follows from Lemma 4 and Proposition 13. □

We have constrained efficiency for the Poisson distribution and for the negative binomial family of distributions because they are invariant search technologies. However, we do not necessarily have efficiency for any general search technology P_k, but only for those which satisfy condition (52). Given that there is no reason why these two terms should be equal to each other, it is difficult to see which natural assumptions could guarantee this condition for efficiency, other than just assuming invariance.

In Appendix B, we present an example of a search technology which is not invariant and does not deliver constrained efficiency of seller entry. For this search technology, there is equilibrium *under-entry* of sellers relative to the social optimum. For this example, the equilibrium surplus share of sellers is $1 - \alpha(\lambda) = e^{-\lambda}$, but the planner wants the optimal surplus share for sellers, which is $\eta_m(\lambda) = 1/(1+\lambda)$. In general, we have $e^{-\lambda} < 1/(1+\lambda)$ for $\lambda > 0$, so sellers are getting a lower surplus share than is optimal and there is under-entry. We can also find examples of search technologies which deliver over-entry of sellers relative to the social optimum.

6.3 Optimal search technology

Suppose that the search technology P_k is invariant. Assume that the planner takes the degree of competition λ as given but can choose the search technology, with its implied degree of competitive dispersion for any given λ. The social planner chooses the search technology P_k to maximize the total surplus per buyer:

$$W(\lambda; P_k) = m(\lambda; P_k)(v - c) - \kappa\,\lambda.$$

The solution to this planner's problem is the Poisson search technology, which minimizes competitive dispersion and maximizes welfare $W(\lambda; P_k)$. This result is similar to the result in Cai et al. (2025) regarding the Poisson (urn-ball) being surplus-maximizing if sellers are homogeneous because it maximizes the number of trades.

Proposition 14. *If P_k is an invariant search technology with mean λ, then*

1. *Competitive dispersion is minimized if and only if P_k is Poisson.*
2. *$m(\lambda; P_k)$ and $W(\lambda; P_k)$ are maximized if and only if P_k is Poisson.*
3. *Competitive dispersion is constrained efficient if and only if P_k is Poisson.*

Given that equilibrium outcomes like markups and price dispersion depend only on the search technology and the degree of competition λ (taking v and c as given), Corollary 5 and Proposition 14 imply that equilibrium markups and price dispersion are socially optimal if and only if the search technology is Poisson.

Corollary 6. *If P_k is an invariant search technology, then equilibrium markups and equilibrium price dispersion are constrained efficient if and only if P_k is Poisson.*

For example, if the search technology is negative binomial but not Poisson, then equilibrium markups and price dispersion are *not* efficient from the perspective of this extended planner's problem. For example, if the search technology is geometric, the aggregate markup may be either too high or too low relative to the social optimum, and price dispersion may be either too high or too low relative to the social optimum.

We can think of the excess competitive dispersion arising from search technologies other than the optimal Poisson as a form of "misallocation". This is a kind of misallocation which is possible even though firms are identical and there are no differences in either productivity or marginal cost. It is a form of misallocation not in the way that resources are distributed across firms, but in the way that *seller competition* is distributed unequally across buyers according to the search technology. The planner would ideally prefer all buyers to experience the same degree of seller competition in order to maximize the number of trades. The Poisson distribution comes closest to this by minimizing competitive dispersion within the invariant class.

Given the Poisson is socially optimal, we might wonder whether the Poisson minimizes price dispersion. Surprisingly, Corollary 7 says that this is not the case.

Corollary 7. *If P_k is an invariant search technology, then minimal competitive dispersion is socially optimal but minimal price dispersion is not always socially optimal.*

Proof. The first part follows from Proposition 14, and the second part follows from Propositions 11 and 12, which imply that the Poisson search technology does not always minimize price dispersion. $\square$

Within the class of invariant search technologies, the Poisson distribution is indeed the unique distribution that minimizes competitive dispersion. However, the Poisson search technology does not always minimize price dispersion. The planner's choice thus minimizes competitive dispersion but does not minimize price dispersion. There is an optimal level of price dispersion that is given by the Poisson search technology, and this is *not* always the minimum possible price dispersion within this class of search technologies. Any policy which aims to minimize price dispersion (for any given λ) may not be socially optimal and may lead to a strict decrease in welfare.

6.4 Welfare loss from competitive dispersion

To get a sense of the welfare loss from excess competitive dispersion above the planner's optimal level (i.e. above the Poisson level), we consider an example. We first consider the short-run effects of competitive dispersion on equilibrium outcomes when the degree of competition λ is exogenous, and then consider the long-run effects when seller entry is determined by a zero profit condition and λ is endogenous. We focus on the negative binomial family of search technologies where $cv^2(\lambda) = 1/\lambda + 1/r$.

	Poisson	$r = 5$	$r = 2$	Geom	$r = 1/2$	$r = 1/3$
Degree of competition, λ	2.00	2.00	2.00	2.00	2.00	2.00
Dispersion parameter, $1/r$	0.00	0.20	0.50	1.00	2.00	3.00
Competitive dispersion, cv	0.71	0.84	1.00	1.22	1.58	1.87
Welfare, W	0.594	0.543	0.479	0.396	0.282	0.207
Consumer surplus	0.594	0.548	0.500	0.444	0.374	0.328
Aggregate markup, $\tilde{\mu}$	1.313	1.326	1.333	1.333	1.324	1.312
Price dispersion, cv_H	0.147	0.153	0.159	0.167	0.176	0.182

Table 1: Short-run equilibrium with $\lambda = 2$, $v = 2$, $c = 1$, $b = 1$, and $\kappa = e^{-2}$.

Table 1 presents the aggregate outcomes when the degree of competition λ is exogenous. Welfare is highest for the Poisson search technology. Price dispersion is lowest for the Poisson search technology for this example (although this is not always true). The aggregate markup varies *non-monotonically* with the degree of competitive dispersion. At first, the aggregate markup is increasing in competitive dispersion, but it later falls. This illustrates the non-monotonicity result in Proposition 10.

Table 2 considers the same example except that the degree of competition is endogenized according to expression (49) for equilibrium seller entry, λ^*.[9] In contrast

[9] In Table 2, competitive dispersion is $cv(\lambda^*)$, the coefficient of variation of the distribution $P_k(\lambda^*)$,

	Poisson	$r = 5$	$r = 2$	Geom	$r = 1/2$	$r = 1/3$
Degree of competition, λ^*	2.00	1.98	1.90	1.72	1.40	1.16
Dispersion parameter, $1/r$	0.00	0.20	0.50	1.00	2.00	3.00
Competitive dispersion, $cv(\lambda^*)$	0.71	0.84	1.01	1.26	1.65	1.97
Welfare, W	0.594	0.543	0.480	0.400	0.298	0.236
Consumer surplus	0.594	0.543	0.480	0.400	0.298	0.236
Aggregate markup, $\tilde{\mu}$	1.313	1.330	1.348	1.368	1.389	1.399
Price dispersion, cv_H	0.147	0.153	0.158	0.162	0.165	0.166

Table 2: Long-run equilibrium with λ endogenous, $v = 2$, $c = 1$, $b = 1$, and $\kappa = e^{-2}$.

to Table 1, which isolates the direct effect of competitive dispersion while holding the degree of competition fixed (as in our results for Section 5), Table 2 incorporates both the direct effect of competitive dispersion and the indirect effect of competitive dispersion through its effect on equilibrium seller entry λ^* in the long run.

	(1) short run (λ exog.)	(2) long run (λ^* endog.)
Welfare, W	−4.62%	−4.55%
Consumer surplus	−4.34%	−4.55%
Aggregate markup, $\tilde{\mu}$	+0.63%	+0.72%
Price dispersion, cv_H	+2.26%	+2.18%

Table 3: Effects of 10% increase in competitive dispersion cv relative to the Poisson.

Table 3 summarizes the welfare loss from a 10% increase in the degree of competitive dispersion from the efficient (Poisson) benchmark, i.e. a 10% increase in the coefficient of variation cv of the distribution $P_k(\lambda)$ of the number of sellers each buyer meets.[10] We use the same parameter values as in Tables 1 and 2 for columns (1) and (2) respectively. A 10% increase in competitive dispersion from the efficient (Poisson) benchmark leads to a 4.62% decrease in welfare, a 0.63% increase in the aggregate markup, and a 2.26% increase in price dispersion for this example. The welfare effects are similar in the long run when seller entry adjusts and λ^* is endogenous.

which varies with both the dispersion parameter $1/r$ and the endogenous value of λ^*.

[10]To implement the 10% increase in competitive dispersion in Table 3, we restrict attention to the negative binomial family $P_k(\lambda; r)$, with the Poisson benchmark given by the limit $r \to \infty$. In this family, $cv^2(\lambda; r) = 1/\lambda + 1/r$, so $cv(\lambda; \infty) = 1/\sqrt{\lambda}$. In column (1), we hold $\lambda = 2$ fixed and choose r so that $cv(2; r) = 1.1 \times cv(2; \infty)$, which implies $r \simeq 9.52$. In column (2), we determine $\lambda^* = \lambda^*(r)$ from the zero profit condition (47) and choose r so that $cv(\lambda^*(r); r) = 1.1 \times cv(\lambda^*_P; \infty)$, where λ^*_P is the Poisson value, which is $\lambda^*_P = 2$ under our chosen parameters, and this yields $r \simeq 9.67$.

7 Buyer heterogeneity

In this paper, we have assumed that the search technology P_k is a primitive. We can indeed treat the search technology as a primitive that captures search frictions, but in some situations we may want to microfound it in terms of heterogeneity in the degree of *competitive intensity* faced by different types of buyers. Competitive intensity is a measure of the relative extent of the effective competition for a buyer. Certain types of buyers may tend to have larger or smaller choice sets of sellers. Heterogeneity in competitive intensity may arise due to differences in search intensity, search costs, income, geography, platforms, advertising, network position, and so on.[11]

One of the advantages of our invariance assumption is that any search technology in this class can be microfounded by some distribution of competitive intensity. On the other hand, any distribution of competitive intensity microfounds some search technology in the invariant class. We describe this one-to-one mapping here.

Microfoundation. Suppose the average number of sellers a buyer meets is λ, but buyers may differ in their *competitive intensity* τ and therefore in the expected number of sellers they meet, i.e. the expected number of price quotes they obtain. The distribution of competitive intensity has mean one and is continuous with cdf Φ and support $[\underline{\tau}, \bar{\tau}] \subseteq \mathbb{R}_+$. In expectation, buyers with $\tau > 1$ meet more sellers than average, while buyers with $\tau < 1$ meet fewer sellers than average.[12]

If a buyer with competitive intensity τ draws a number of price quotes from a distribution P_k^τ that is Poisson with mean $\tau\lambda$, we say that the distribution of competitive intensity Φ "generates" a search technology P_k in the sense of Definition 2. This search technology P_k gives us the (unconditional) distribution of the number of price quotes obtained across *all* buyers in the market.[13]

Definition 2. *A distribution of competitive intensity* Φ *with support* $[\underline{\tau}, \bar{\tau}] \subseteq \mathbb{R}_+$ generates *a search technology* P_k *if and only if, for all* $k \in \mathbb{N}$ *and any* $\lambda > 0$,

$$P_k(\lambda) = \int_0^\infty \frac{(\lambda\tau)^k e^{-\lambda\tau}}{k!} d\Phi(\tau). \tag{53}$$

[11]Note that this interpretation of the search technology in terms of buyer heterogeneity means that the welfare results in Section 6 need no longer apply, depending on the source of heterogeneity.

[12]We also allow the case where Φ is degenerate at $\tau = 1$, i.e. there is no buyer heterogeneity.

[13]The search technology generated is a mixed Poisson distribution and Φ is sometimes called the "mixing distribution". See Cai et al. (2025) and Mangin (2026) for further discussion of this class.

For any distribution of competitive intensity Φ, it is clear that there exists a unique search technology P_k that the distribution generates because we can use (53) to construct this search technology. This is known as a mixed Poisson search technology and it is straightforward to show that it is *invariant*.[14] Perhaps more surprisingly, the converse is also true. Any invariant search technology can be generated by some distribution of competitive intensity. This result was proven in Cai et al. (2025).

Proposition 15. *Any distribution of competitive intensity* Φ *generates a unique invariant search technology* P_k*. Conversely, for any invariant search technology* P_k*, there exists a unique distribution of competitive intensity* Φ *that generates* P_k*.*

If we observe only the distribution of the size of buyers' choice sets, i.e. the distribution of the number of price quotes, then we can determine the unique distribution of competitive intensity that generates it. Conversely, if we start with a distribution of competitive intensity, we can determine the corresponding search technology and then apply any of our results for the invariant class. For example, the Poisson search technology is generated by a degenerate distribution of competitive intensity (i.e. all buyers have the same competitive intensity), while the geometric search technology is generated by an exponential distribution of competitive intensity and the negative binomial search technology is generated by a Gamma distribution.[15]

Our use of invariant search technologies allows us to model buyer heterogeneity, either in search intensity or any other type of heterogeneity (e.g. income) which may lead buyers to differ in the size of their choice sets. This generates heterogeneity in the (conditional) distribution of prices – and thus the average price and the price dispersion – faced by buyers. For example, suppose the search technology P_k is geometric. Equivalently, the distribution of competitive intensity Φ is exponential. In this case, the distribution of posted prices F is given by (107) for the geometric example. However, the (conditional) distribution of transaction prices $H(p|\tau)$ for a buyer with competitive intensity τ is the same as (33) for the Poisson example with mean equal to $\lambda\tau$, except that the distribution of posted prices F is now different.

In fact, for *any* invariant search technology P_k, the (conditional) distribution of transaction prices $H(p|\tau)$ for a buyer with competitive intensity τ takes the same form – except the distribution F is different. For all buyers, the distribution F of

[14] We verify this in the proof of Proposition 15 in the Appendix.

[15] For examples of invariant search technologies and their corresponding distributions of "types," see the examples in Cai et al. (2025) or the table in Appendix E of Mangin (2026).

posted prices is given by Proposition 4, the aggregate markup is given by Proposition 2, and overall price dispersion is given by Proposition 3. However, the (conditional) distribution of prices $H(p|\tau)$ for a buyer of type τ is described by our next result.

Proposition 16. *Suppose P_k is an invariant search technology that is generated by a distribution of competitive intensity Φ with support $[\underline{\tau}, \bar{\tau}] \subseteq \mathbb{R}_+$. For a buyer with competitive intensity $\tau > 0$, the (conditional) distribution of transaction prices is*

$$H(p|\tau) = \frac{1 - e^{-\lambda\tau F(p)}}{1 - e^{-\lambda\tau}} \tag{54}$$

where the equilibrium distribution of posted prices F is

$$F(p) = \frac{1}{\lambda}(Q_1)^{-1}\left(Q_1(\lambda)\frac{v-c}{p-c}\right) \tag{55}$$

and $Q_1(\lambda)$ is the probability a seller has no competitors, i.e. has a "captive" buyer.

Proof. For a buyer with competitive intensity $\tau > 0$, the distribution P_k^τ of the number of sellers they meet is Poisson with mean $\tau\lambda$. The expression for $H(p|\tau)$ follows from (33) for the Poisson with mean $\lambda\tau$. However, the distribution of posted prices F is different from the Poisson case: it is given by (18) in Proposition 4. □

Importantly, the (conditional) distribution of transaction prices $H(p|\tau)$ for a buyer with competitive intensity τ depends on both $\lambda\tau$ *and* the equilibrium distribution of posted prices F. The effects of competitive intensity τ on the average price and on price dispersion are *not* the same as the effects of λ because of this dependence of $H(p|\tau)$ on the distribution F, which depends on the search technology P_k.

Corollary 8 summarizes the effects of competitive intensity on the distribution of the number of price quotes P_k^τ, the distribution of transaction prices $H(p|\tau)$, the average transaction price $\tilde{p}(\lambda|\tau) \equiv \mathbb{E}_{H(\cdot|\tau)}[p]$, and the price dispersion $cv_H(\lambda|\tau)$. For any invariant search technology, greater competitive intensity leads to more price quotes and lower transaction prices on average. The average price paid by a buyer is decreasing in the buyer's competitive intensity τ, and the price dispersion experienced by a buyer converges to zero as the buyer's competitive intensity becomes large.

Corollary 8. *Suppose that P_k is an invariant search technology that is generated by a distribution of competitive intensity Φ with support $[\underline{\tau}, \bar{\tau}] \subseteq \mathbb{R}_+$.*

1. *If $\tau_2 > \tau_1$, the distribution of the number of quotes $P_k^\tau(\lambda\tau_1) \preccurlyeq_{FOSD} P_k^\tau(\lambda\tau_2)$.*

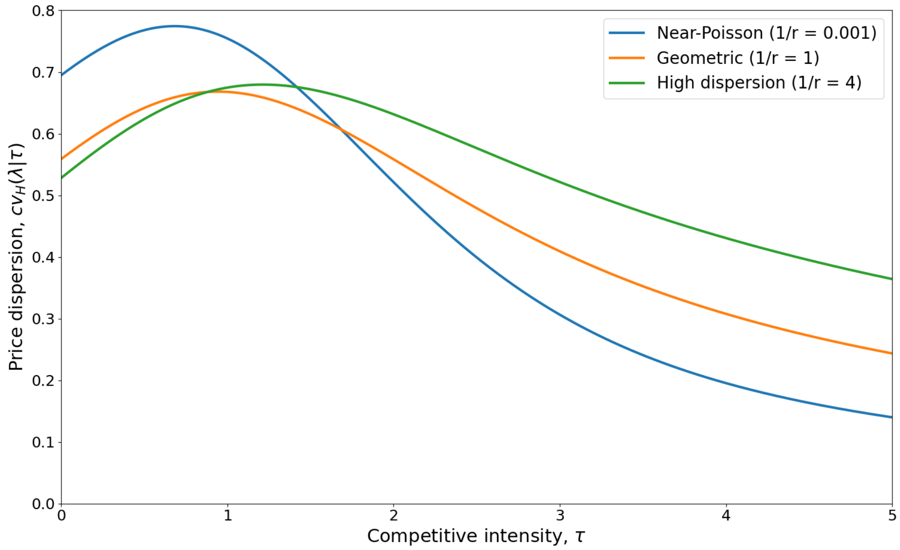

Figure 8: Price dispersion and competitive intensity, NB family ($\lambda = 3$, $c = 0.05$, $v = 1$)

2. *If $\tau_2 > \tau_1$, the distribution of transaction prices $H(\cdot, \lambda\tau_2) \preccurlyeq_{FOSD} H(\cdot, \lambda\tau_1)$.*

3. *The average price paid is decreasing in a buyer's competitive intensity τ.*

4. *If $\bar{\tau} = \infty$, then price dispersion $cv_H^2(\lambda|\tau) \to 0$ in the limit as $\tau \to \infty$.*

5. *If $\underline{\tau} = 0$, then price dispersion $cv_H^2(\lambda|\tau) \to cv_F^2(\lambda)$ in the limit as $\tau \to 0$.*

If the distribution of competitive intensity τ has unbounded upper support, then price dispersion converges to zero in the limit as τ becomes large. However, price dispersion need not be strictly decreasing in a buyer's competitive intensity, but can instead be non-monotonic. Figure 8 illustrates some non-monotonic examples.[16] As the figure shows, buyers with higher competitive intensity do not necessarily experience lower price dispersion, although they do pay lower prices on average.

8 Application: Labor market

In the labor market, competitive dispersion represents dispersion in the degree of effective competition for workers. This may be captured by dispersion in the number of job offers a worker receives or the number of firms in a worker's "choice set". In

[16] For the Poisson, Φ is degenerate at $\tau = 1$, so we take $1/r = 0.001$ instead of the Poisson limit.

this section, we present a simple application of our results to the labor market. We ask: How does competitive dispersion in the labor market affect wages? How does competitive dispersion affect equilibrium wage dispersion?

As in Sections 2-6, we assume that the search technology P_k, which determines the distribution of the number of job offers a worker receives, is a primitive. We also assume that this search technology is invariant. However, as shown in Section 7, any invariant search technology can be generated by a distribution of worker search intensity, which may arise due to heterogeneity across workers, via Definition 2.

8.1 Environment

The environment is the same as in Section 2 except that we interpret the "sellers" as *firms* that are searching for workers, and the "buyers" as *unemployed workers*. A worker who does not receive (or does not accept) an offer remains unemployed.

There is a continuum of measure U of unemployed workers and a continuum of measure V of firms. The *labor market tightness* is $\theta \equiv V/U$. Workers have an outside option $z \geq 0$, which represents the flow value of unemployment. A filled job produces output $y > z$. Workers obtain flow payoff w if they are employed at wage w, and z if unemployed. A vacancy that hires a worker at wage w obtains profit $y - w$.

Let $P_k(\theta)$ denote the probability that a worker receives $k \in \mathbb{N}$ wage offers. Let $Q_k(\theta)$ denote the probability that a vacancy is in a meeting with k vacancies (i.e. the worker it meets receives k job offers including its own). As before, consistency implies the relationship $\theta Q_k(\theta) = k P_k(\theta)$ for all $k \in \mathbb{N}$, which corresponds to (1).

Workers accept the highest wage offer w they receive provided that $w \geq z$. Define the *match profit* as $p \equiv y - w$. A worker who accepts wage w obtains surplus $w - z = y - p - z$, and thus chooses the lowest profit p among their offers. After setting $p = y - w$, the labor market environment is isomorphic to the Burdett–Judd environment in Section 2 with "marginal cost" $c = 0$, buyer "valuation" $v = y - z$, and "degree of competition" $\lambda = \theta$, the labor market tightness.

8.2 Wage distribution

Let F_w denote the equilibrium cdf of posted wages, with density f_w. Let $q_w(w)$ denote the probability a vacancy posting wage w hires a worker (i.e. the "quantity" of workers hired). Let $\tilde{q}(\theta) \equiv m(\theta)/\theta$ denote the average "quantity" of workers hired per vacancy, where $m(\theta) = 1 - P_0(\theta)$ is the workers' matching probability.

Proposition 17. *For any market tightness $\theta > 0$, there exists a unique symmetric mixed-strategy equilibrium. The equilibrium distribution of posted wages has no mass points and cdf F_w with support $w \in [z, \bar{w}]$, where $\bar{w} = y - Q_1(\theta)(y - z)$.*

If P_k is invariant, the distribution of posted wages F_w is given by

$$F_w(w) = 1 - \frac{1}{\theta}(Q_1)^{-1}\left(Q_1(\theta)\frac{y-z}{y-w}\right). \tag{56}$$

Equivalently, any posted wage w satisfies the wage condition:

$$w = z + \left(1 - \frac{Q_1(\theta)}{q_w(w)}\right)(y - z) \tag{57}$$

where $q_w(w)$ is the "quantity" of workers hired at wage w:

$$q_w(w) = Q_1(\theta(1 - F_w(w))) \tag{58}$$

and $Q_1(\theta)$ is the probability a firm has no competitors, i.e. a monopsony hire.

Let H_w denote the cdf of the distribution of accepted wages. We measure wage dispersion using the coefficient of variation $cv_{H_w}(\theta)$ of the distribution of accepted wages. That is, wage dispersion is given by $cv^2_{H_w}(\theta) \equiv \sigma^2_{H_w}(\theta)/\tilde{w}(\theta)^2$.

Lemma 8. *If P_k is invariant with mean θ, the distribution of accepted wages is*

$$H_w(w) = \frac{P_0(\theta(1 - F_w(w))) - P_0(\theta)}{1 - P_0(\theta)}. \tag{59}$$

Equilibrium wage dispersion is given by

$$cv^2_{H_w}(\theta) = \left(\frac{\mathbb{E}_{F_w}[y - w]}{y - \tilde{w}(\theta)} - 1\right)\left(\frac{y - \tilde{w}(\theta)}{\tilde{w}(\theta)}\right)^2. \tag{60}$$

Let $\tilde{w}(\theta) \equiv \mathbb{E}_{H_w}[w]$, the average wage paid for accepted job offers. Define the share of *monopsony hires* by $1 - \alpha(\theta) \equiv Q_1(\theta)/\tilde{q}(\theta)$. This is the share of hires occurring when the hiring vacancy has no competitors. The share of *competitive hires*, where there was direct competition to hire the worker, is equal to $\alpha(\theta)$.

If P_k is invariant, the share of monopsony hires is equal to the matching elasticity, $\eta_m(\theta)$. As we have seen in Section 6, this is why invariance delivers constrained efficiency via the Hosios condition because it means that firms' surplus share is equal to the matching elasticity. Given this, it is clear that the average wage $\tilde{w}(\theta)$ is strictly increasing in market tightness θ if and only if $\eta_m(\theta)$ is strictly decreasing in θ.

Proposition 18. *For any market tightness $\theta > 0$, the average wage is*

$$\tilde{w}(\theta) = z + \alpha(\theta)(y - z) \tag{61}$$

where $\alpha(\theta)$ is the share of competitive hires. If P_k is invariant, then

$$\tilde{w}(\theta) = z + (1 - \eta_m(\theta))\,(y - z)\,. \tag{62}$$

Proof. Let $p \equiv y - w$ be match profit. Then $\tilde{w}(\theta) = y - \tilde{p}(\theta)$, where $\tilde{p}(\theta) \equiv \mathbb{E}_H\,[p]$ is the average accepted profit. Applying Proposition 2 with $c = 0$ and $v = y - z$ yields (61). If P_k is invariant, Lemma 4 implies $1 - \alpha(\theta) = \eta_m(\theta)$, which yields (62). $\square$

Figure 9 depicts some examples of how the average wage varies with the labor market tightness for the negative binomial family of search technologies. In all cases, the average wage is increasing in market tightness. However, for the geometric example and the highest dispersion example ($1/r = 4$), the average wage increases more sharply at first compared to the Poisson (for lower levels of market tightness) but also flattens significantly later on (for higher levels of market tightness). For the Poisson, the average wage increases with market tightness at a relatively constant rate.

8.3 Effect of competitive dispersion

Competitive dispersion in the labor market corresponds to dispersion in the number of wage offers workers receive, holding market tightness θ fixed. Our measure of competitive dispersion is again the coefficient of variation $cv(\theta)$ of the distribution $P_k(\theta)$, which is given by Lemma 7 for any invariant search technology.

As in Section 5, we consider the negative binomial family and vary competitive dispersion by varying the parameter $1/r$ while holding the market tightness θ fixed.

Next, we ask: How does competitive dispersion affect the average wage? Again, we consider the negative binomial family of search technologies. Similarly to Proposition 10, we find that the direction of the effect of competitive dispersion on the average wage depends on both the search technology and the labor market tightness.

We also ask: How does competitive dispersion affect unemployment? In equilibrium, workers are hired provided they receive at least one job offer, so the unemploy-

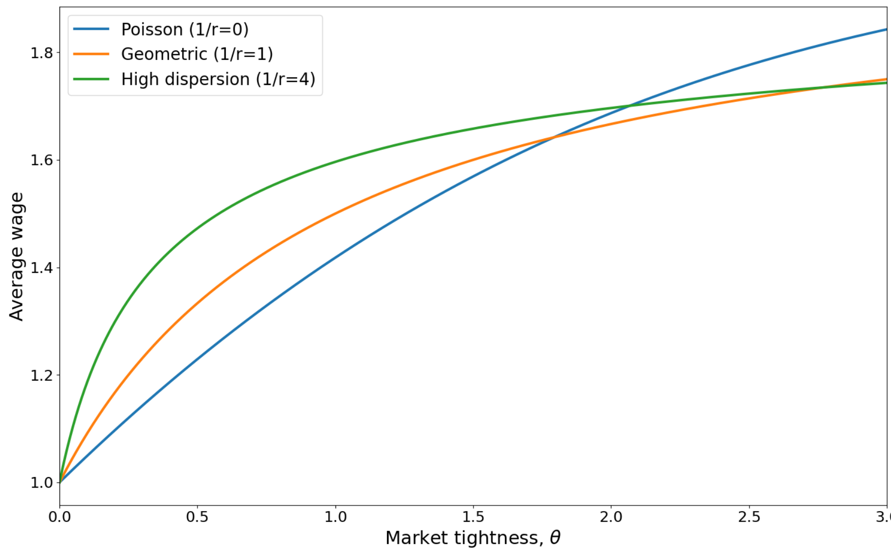

Figure 9: Average wage and labor market tightness, NB family ($z = 1$ and $y = 2$)

ment rate is $u(\theta) = 1 - m(\theta) = P_0(\theta)$, i.e. the probability that a worker receives no job offers. We find that unemployment is strictly increasing in competitive dispersion.

Proposition 19. *Suppose that P_k is negative binomial with mean θ and competitive dispersion $1/r$. Let $\theta_d^* \approx 1.59$ be the unique solution to $e^\theta = 2/(2-\theta)$.*

1. *The unemployment rate $u(\theta)$ is strictly increasing in competitive dispersion.*
2. *If $\theta < \theta_d^*$, the average wage is strictly increasing in competitive dispersion.*
3. *If $\theta > \theta_d^*$, there exists a unique $r^*(\theta) \in (0, \infty)$ such that the average wage is decreasing in dispersion on $(0, 1/r^*(\theta))$ and increasing in dispersion on $(1/r^*(\theta), \infty)$.*

Finally, it is difficult to obtain general comparative statics results regarding the effect of competitive dispersion $1/r$ on wage dispersion, so our next result describes the *local effect* of competitive dispersion on equilibrium wage dispersion around the Poisson benchmark for the special case where $z = 0$. This case is sufficient to determine that higher competitive dispersion may either increase or decrease wage dispersion.

Proposition 20. *Suppose that P_k is negative binomial with mean θ and competitive dispersion $1/r$. Assume that workers' outside option is $z = 0$. There exists a unique cutoff $\theta_z^* \approx 0.91$ such that, in a neighborhood of $1/r = 0$ (Poisson),*

1. *If $\theta < \theta_z^*$, wage dispersion is strictly decreasing in competitive dispersion.*

2. *If $\theta > \theta_z^*$, wage dispersion is strictly increasing in competitive dispersion.*

Greater competitive dispersion does not necessarily increase wage dispersion, which is surprising and counter-intuitive. The direction of the effect depends on the search technology, the labor market tightness, and workers' outside option.

9 Conclusion

This paper introduces the notion of *competitive dispersion*, which represents dispersion in the effective competition for different buyers. We study the effect of competitive dispersion on both markups and price dispersion in a Burdett-Judd search-theoretic model of imperfect competition. We find that greater competitive dispersion can either increase *or* decrease the aggregate markup, depending on both the search technology and the degree of competition. Surprisingly, while competitive dispersion is necessary for price dispersion, greater competitive dispersion does *not* always lead to higher price dispersion, but may sometimes decrease price dispersion. Seller entry is constrained efficient whenever the search technology is invariant. However, if the planner can choose the search technology, then equilibrium markups and price dispersion are constrained efficient if and only if the search technology is Poisson, which minimizes competitive dispersion but not necessarily price dispersion. If the search technology is not Poisson, excess competitive dispersion can be seen as a kind of "misallocation" of competition across buyers which reduces welfare – although the interpretation of this welfare result depends on how exactly the dispersion arises.

We consider a simple application of our results to the labor market. In a labor market setting, competitive dispersion represents dispersion in the degree of effective competition for workers. Greater competitive dispersion always leads to higher unemployment. However, it can either increase or decrease the average wage, depending on both the search technology and the labor market tightness. Similarly, greater competitive dispersion may either increase or decrease wage dispersion, depending on the search technology, the labor market tightness, and workers' outside option.

This paper has focused on the case of homogeneous firms because the effects of competitive dispersion are sufficiently interesting even in this extreme case where all firms have the same marginal cost and produce identical goods for which all buyers have the same valuation. This has allowed us to highlight the role played by the search technology, i.e. the distribution of the number of price quotes. In future work, however, it would be interesting to extend the model to the case where buyers have idiosyncratic valuations for the goods produced by different firms, or the case of heterogeneous firms where there is a distribution of firms' marginal costs. Another direction for future research would be to endogenize the distribution of competitive intensity based on buyer heterogeneity – in terms of search effort, geographic location, or income – in order to study how markups vary across the income distribution or across locations and to determine the welfare consequences of this heterogeneity.

Appendix A: Proofs

A.1 Proofs for Section 3

Proof of Proposition 1

Proof. We first show that F has no mass points and its support is an interval $[\underline{p}, v]$.

Suppose, for the sake of contradiction, that F has a mass point at some $p_0 \in (c, v]$. Consider a seller who deviates from p_0 to $p_0 - \varepsilon$ for $\varepsilon > 0$. For any meeting size $k \geq 2$, with strictly positive probability at least one rival posts exactly p_0 and no rival posts a price below p_0. In this event, a seller posting p_0 (at best) ties with those rivals, while the deviator wins the sale. Hence the deviator's probability of sale increases by a strictly positive amount that does not vanish as $\varepsilon \to 0$, while its profit margin falls by only ε. For ε small enough, this deviation strictly increases expected profit, contradicting seller optimality. Therefore, F has no mass points.

Now suppose, for the sake of contradiction, that the support of F is not an interval. Then there exist $p_0 < p_1$ such that F is constant on (p_0, p_1). Since the probability of sale depends on p only through $F(p)$, it follows that the quantity demanded $q(p)$ is constant on (p_0, p_1). A seller posting p_0 can therefore profitably deviate to $p_0 + \varepsilon \in (p_0, p_1)$, which strictly increases its profit without affecting its probability of sale, contradicting seller optimality. Hence the support of F is an interval $[\underline{p}, \bar{p}]$.

Next, we have $\bar{p} \leq v$ clearly. Now suppose, for the sake of contradiction, that $\bar{p} < v$. Since F is supported on $[\underline{p}, \bar{p}]$, all other sellers post prices at most $\bar{p}$ almost surely. Hence any seller who posts any price $p \in (\bar{p}, v]$ can make a sale only when it faces no rivals, i.e. only when $k = 1$. Therefore its probability of sale is $Q_1(\lambda)$, independent of the choice of $p \in (\bar{p}, v]$. But then posting v yields strictly higher profit than posting any $p < v$, contradicting that $\bar{p}$ is the top of the support. Thus $\bar{p} = v$.

We now derive the equilibrium expressions in Proposition 1. If a buyer meets k sellers, a seller posting p trades if and only if all $k-1$ rivals post prices above p, which occurs with probability $(1 - F(p))^{k-1}$. Therefore, since $Q_k(\lambda)$ is the probability a seller has $k-1$ rivals, the quantity demanded for a seller posting price p is given by

$$q(p) = \sum_{k=1}^{\infty} Q_k(\lambda)(1 - F(p))^{k-1}.$$

The expected profit for a seller from posting price p is

$$\pi(p) = (p-c)q(p) = (p-c)\sum_{k=1}^{\infty} Q_k(\lambda)(1-F(p))^{k-1}.$$

In a seller-symmetric mixed-strategy equilibrium, sellers are indifferent across all p in the support, so $\pi(p)$ is constant on $[\underline{p}, \bar{p}]$. Since $F(v) = 1$, we obtain $\pi(v) = Q_1(\lambda)(v-c)$. Since profit is constant for all posted p, we have

$$\pi(p) = \tilde{\pi}(\lambda) \equiv Q_1(\lambda)(v-c) \tag{63}$$

for *all* posted prices p. Finally, $F(\underline{p}) = 0$ so $q(\underline{p}) = 1$, thus $\underline{p} = c + Q_1(\lambda)(v-c)$. □

Proof of Proposition 2

Proof. To start with, we have the following:

$$\mathbb{E}_H[p-c] = \int (p-c)h(p)dp = \frac{\lambda}{1-P_0(\lambda)} \int (p-c)q(p)f(p)dp, \tag{64}$$

using (6) for $h(p)$. The equilibrium pricing condition and $Q_1(\lambda) = P_1(\lambda)/\lambda$ imply

$$\pi(p) = (p-c)q(p) = \frac{P_1(\lambda)(v-c)}{\lambda}. \tag{65}$$

Substituting the above into (64) and simplifying yields (8) because $\int f(p)dp = 1$.

Finally, letting $1-\alpha(\lambda)$ denote the probability a buyer meets only one seller, conditional on purchasing, we have the following:

$$1-\alpha(\lambda) = \frac{P_1(\lambda)}{1-P_0(\lambda)}. \tag{66}$$

Substituting into (8) and rearranging yields the weighted average (9). □

Proof of Proposition 3

Proof. Substituting equation (10) into (6) delivers the following expression:

$$h(p) = \frac{\tilde{p}(\lambda)-c}{p-c} f(p). \tag{67}$$

Since h is a density, integrating (67) over the support gives

$$1 = \int h(p)\,dp = (\tilde{p}(\lambda) - c) \int \frac{1}{p-c} f(p)\,dp, \tag{68}$$

so we have

$$\tilde{p}(\lambda) - c = \left(\mathbb{E}_F[(p-c)^{-1}]\right)^{-1}. \tag{69}$$

Next, using (67), we have

$$\mathbb{E}_H\left[p^2\right] = \int p^2 h(p)\,dp = (\tilde{p}(\lambda) - c) \int \frac{p^2}{p-c} f(p)\,dp.$$

Rewriting p as $p = (p-c) + c$, we obtain

$$\frac{p^2}{p-c} = \frac{((p-c)+c)^2}{p-c} = (p-c) + 2c + \frac{c^2}{p-c}.$$

Combining the above two results yields

$$\mathbb{E}_H\left[p^2\right] = (\tilde{p}(\lambda) - c)\left(\mathbb{E}_F\left[p-c\right] + 2c + c^2 \int \frac{1}{p-c} f(p)\,dp\right).$$

Using (69), this becomes

$$\mathbb{E}_H\left[p^2\right] = (\tilde{p}(\lambda) - c)\,\mathbb{E}_F\left[p-c\right] + 2c\left(\tilde{p}(\lambda) - c\right) + c^2. \tag{70}$$

By definition, $\mathbb{E}_H\left[p\right] = \tilde{p}(\lambda)$ and simplifying yields

$$cv_H^2(\lambda) = \frac{\mathbb{E}_H\left[p^2\right]}{\mathbb{E}_H\left[p\right]^2} - 1 = \frac{(\tilde{p}(\lambda) - c)\,\mathbb{E}_F\left[p-c\right] + 2c\tilde{p}(\lambda) - c^2}{\tilde{p}(\lambda)^2} - 1.$$

Rearranging the above gives us the desired expression (11). □

Proof of Lemma 2

Proof. It is useful to first state the following lemma derived in Cai et al. (2025), which uses the fact that if P_k is invariant then the function P_0 is completely monotone to provide a Laplace transform representation for P_0. See Appendix E in Mangin (2026).

Lemma 9. *If P_k is an invariant search technology with mean $\lambda > 0$, then*

$$P_0(\lambda) = \int_0^\infty e^{-\lambda\tau} d\Phi(\tau) \tag{71}$$

for some probability distribution with cdf Φ *and mean* $\mathbb{E}_{\Phi}[\tau] = 1$.

We can use this result to prove parts of Lemma 2. We can also use the fact that, by Definition 1, the probability generating function of P_k is given by

$$\mathbb{G}(z) \equiv \sum_{k=0}^{\infty} P_k(\lambda) z^k = P_0(\lambda(1-z)). \tag{72}$$

(i) and *(ii)*. Differentiating expression (71) yields $P_0'(\lambda) < 0$ and $P_0''(\lambda) > 0$.

(iii). Because $\mathbb{G}(1) = 1$, invariance implies $P_0(0) = 1$, and thus $\lim_{\lambda \to 0} P_0(\lambda) = 1$.

(iv). By Lemma 9, $P_0(\lambda)$ has a Laplace transform representation $P_0(\lambda) = \mathbb{E}_{\Phi}\left[e^{-\lambda\tau}\right]$ for some distribution Φ with $\mathbb{E}_{\Phi}[\tau] = 1$. The Laplace transform converges to $\Pr(\tau = 0)$ as $\lambda \to \infty$, implying $\lim_{\lambda \to \infty} P_0(\lambda) = 0$ if and only if $\Pr(\tau = 0) = 0$. In Section 2, we assume that $\lim_{\lambda \to \infty} P_0(\lambda) = 0$, which is equivalent to assuming $\Pr(\tau = 0) = 0$.

(v). Differentiating $P_0(\lambda(1-z))$ and evaluating at $z = 1$ yields $\mathbb{G}'(1) = -\lambda P_0'(0)$. Since $\mathbb{G}'(1) = \sum_{k=1}^{\infty} k P_k(\lambda) = \lambda$, we have $\lim_{\lambda \to 0} P_0'(\lambda) = -1$.

(vi). Differentiating $P_0(\lambda) = \mathbb{E}_{\Phi}\left[e^{-\lambda\tau}\right]$ yields $P_0'(\lambda) = -\mathbb{E}_{\Phi}\left[\tau e^{-\lambda\tau}\right]$. By dominated convergence, $\tau e^{-\lambda\tau} \to 0$ pointwise and is bounded by τ, so $\lim_{\lambda \to \infty} P_0'(\lambda) = 0$.

(vii). Using $Q_k(\lambda) = k P_k(\lambda)/\lambda$ and the identity $P_1(\lambda) = \mathbb{G}'(0)$, invariance implies $Q_1(\lambda) = -P_0'(\lambda)$ for all $\lambda > 0$. The limits for Q_1 follow from parts (v) and (vi). $\square$

Proof of Lemma 3

Proof. Suppose that P_k is invariant. By Definition 1, for all $z \in [0, 1]$, we have

$$\sum_{k=0}^{\infty} P_k(\lambda) z^k \; = \; P_0(\lambda\,(1-z))\,. \tag{73}$$

Differentiate (73) with respect to z to obtain

$$\sum_{k=1}^{\infty} k P_k(\lambda) z^{k-1} = -\lambda P_0'(\lambda\,(1-z))\,. \tag{74}$$

Next, using $\lambda Q_k(\lambda) \equiv k P_k(\lambda)$, we have

$$\sum_{k=1}^{\infty} Q_k(\lambda)\, z^{k-1} = -P_0'(\lambda\,(1-z))\,. \tag{75}$$

Finally, by Lemma 2 part (vii), invariance implies $Q_1(x) = -P_0'(x)$ for all $x > 0$. Substituting this identity into (75) yields

$$\sum_{k=1}^{\infty} Q_k(\lambda) z^{k-1} = Q_1(\lambda(1-z)). \tag{76}$$

Now assume that (76) holds. Let $y \equiv 1 - z$. Then (76) can be rewritten as

$$Q_1(\lambda y) = \sum_{k=1}^{\infty} Q_k(\lambda)\,(1-y)^{k-1}. \tag{77}$$

Next, integrate (77) from $y = 0$ to $y = 1$. The left-hand side is

$$\int_0^1 Q_1(\lambda y)\ dy = \frac{1}{\lambda}\int_0^{\lambda} Q_1(u)\,du.$$

The right-hand side is

$$\sum_{k=1}^{\infty} Q_k(\lambda) \int_0^1 (1-y)^{k-1} dy = \sum_{k=1}^{\infty} \frac{Q_k(\lambda)}{k} = \frac{1}{\lambda}\sum_{k=1}^{\infty} P_k(\lambda) = \frac{1-P_0(\lambda)}{\lambda},$$

where we used $\lambda Q_k(\lambda) = kP_k(\lambda)$ and $\sum_{k=0}^{\infty} P_k(\lambda) = 1$. Equating both sides yields

$$1 - P_0(\lambda) = \int_0^{\lambda} Q_1(u)\,du. \tag{78}$$

Next, integrate from 0 to $1-z$. Integrate (77) from $y = 0$ to $y = 1 - z$:

$$\int_0^{1-z} Q_1(\lambda y)\ dy = \int_0^{1-z} \sum_{k=1}^{\infty} Q_k(\lambda)(1-y)^{k-1}\,dy.$$

The left-hand side becomes

$$\int_0^{1-z} Q_1(\lambda y)\ dy = \frac{1}{\lambda}\int_0^{\lambda(1-z)} Q_1(u)\,du = \frac{1}{\lambda}\,(1 - P_0(\lambda(1-z)))$$

where the last equality uses (78). The right-hand side is

$$\sum_{k=1}^{\infty} Q_k(\lambda) \int_0^{1-z} (1-y)^{k-1} dy = \sum_{k=1}^{\infty} Q_k(\lambda)\frac{1-z^k}{k} = \frac{1}{\lambda}\sum_{k=1}^{\infty} P_k(\lambda)\left(1 - z^k\right).$$

Equating both sides and multiplying by λ yields

$$1 - P_0(\lambda(1-z)) = (1 - P_0(\lambda)) - \sum_{k=1}^{\infty} P_k(\lambda) z^k.$$

Rearranging, we obtain

$$P_0(\lambda(1-z)) = P_0(\lambda) + \sum_{k=1}^{\infty} P_k(\lambda) z^k = \sum_{k=0}^{\infty} P_k(\lambda) z^k.$$

Therefore, we have equation (73) as desired. $\square$

A.2 Proofs for Section 4

Proof of Proposition 5

Proof. It is clear from (41) that the elasticity η_{Q_1} of the function Q_1 is equal to the elasticity $\eta_{m'}$ of the function m'. By Assumption 1, $\eta_{Q_1}(\lambda)$ is strictly decreasing for all $\lambda > 0$ and therefore $\eta_{m'}(\lambda)$ is strictly decreasing for all $\lambda > 0$.

1. By Lemma 10 below, we know that if $\eta_{m'}$ is strictly decreasing then η_m is strictly decreasing, i.e. $\eta_m'(\lambda) < 0$ for all $\lambda > 0$. Lemma 5 then implies that the aggregate price and the aggregate markup are strictly decreasing in λ.

2. Fix $x \in (0,1)$. Under invariance, the quantile price given by Corollary 1 is

$$p(x;\lambda) = c + \frac{Q_1(\lambda)}{Q_1(\lambda x)}(v - c), \tag{79}$$

where $Q_1(\lambda) = -P_0'(\lambda)$ by Lemma 2. Since $v > c$, it suffices to show that $Q_1(\lambda)/Q_1(\lambda x)$ is strictly decreasing in λ. Differentiating $\log(Q_1(\lambda)/Q_1(\lambda x))$ yields

$$\begin{aligned}
\frac{\partial}{\partial\lambda}\left[\log Q_1(\lambda) - \log Q_1(\lambda x)\right] &= \frac{Q_1'(\lambda)}{Q_1(\lambda)} - x\frac{Q_1'(\lambda x)}{Q_1(\lambda x)} \\
&= \frac{P_0''(\lambda)}{P_0'(\lambda)} - x\frac{P_0''(\lambda x)}{P_0'(\lambda x)} \\
&= \frac{1}{\lambda}\left(\eta_{m'}(\lambda) - \eta_{m'}(\lambda x)\right) \qquad (80)
\end{aligned}$$

by the definition of $\eta_{m'}$. Because $x \in (0,1)$ implies $\lambda x < \lambda$ and $\eta_{m'}$ is strictly decreasing, the right-hand side of (80) is strictly negative. Hence $Q_1(\lambda)/Q_1(\lambda x)$ is strictly decreasing in λ, and (79) implies that $p(x;\lambda)$ is strictly decreasing in λ.

Therefore, the quantile markup $\mu(x;\lambda)$ is strictly decreasing in λ.

3. Let $\lambda_2 > \lambda_1$. Since $p(\cdot;\lambda)$ is the quantile function of $F(\cdot;\lambda)$, part 2 implies

$$p(x;\lambda_2) < p(x;\lambda_1)$$

for all $x \in (0,1)$, which is equivalent to the result $F(\cdot;\lambda_2) \preccurlyeq_{FOSD} F(\cdot;\lambda_1)$.

4. Under invariance, Corollary 2 tells us that

$$H(p;\lambda) = \frac{1 - P_0(\lambda F(p;\lambda))}{1 - P_0(\lambda)} = \frac{m(\lambda F(p;\lambda))}{m(\lambda)}, \tag{81}$$

where $m(\lambda) \equiv 1 - P_0(\lambda)$. Define

$$T_\lambda(x) \equiv \frac{m(\lambda x)}{m(\lambda)}, \qquad x \in [0,1]. \tag{82}$$

Then $H(p;\lambda) = T_\lambda(F(p;\lambda))$. For each $\lambda > 0$, $T_\lambda(x)$ is strictly increasing in x since

$$\frac{\partial}{\partial x} T_\lambda(x) = \frac{\lambda m'(\lambda x)}{m(\lambda)} > 0.$$

Differentiating $\log T_\lambda(x) = \log m(\lambda x) - \log m(\lambda)$ with respect to λ, we have

$$\begin{aligned} \frac{\partial}{\partial \lambda} \log T_\lambda(x) &= x\frac{m'(\lambda x)}{m(\lambda x)} - \frac{m'(\lambda)}{m(\lambda)} \\ &= \frac{1}{\lambda}\left(\eta_m(\lambda x) - \eta_m(\lambda)\right). \end{aligned} \tag{83}$$

By Lemma 10, if $\eta_{m'}$ is strictly decreasing then η_m is strictly decreasing. Since $\lambda x < \lambda$, the right-hand side of (83) is strictly positive, so $T_\lambda(x)$ is strictly increasing in λ.

Now let $\lambda_2 > \lambda_1$ and fix $p < v$. From part 3, we have $F(p;\lambda_2) \geq F(p;\lambda_1)$. Using the monotonicity of T_λ in both arguments, we have

$$H(p;\lambda_2) = T_{\lambda_2}(F(p;\lambda_2)) \geq T_{\lambda_2}(F(p;\lambda_1)) \geq T_{\lambda_1}(F(p;\lambda_1)) = H(p;\lambda_1).$$

Hence we obtain the desired result that $H(\cdot;\lambda_2) \preccurlyeq_{FOSD} H(\cdot;\lambda_1)$. $\square$

Lemma 10. *If $\eta_{m'}$ is strictly decreasing, then η_m is strictly decreasing.*

Proof. Since $m(0) = 0$ and $m'(\lambda) > 0$, we can write

$$m(\lambda) = \int_0^\lambda m'(s)\,ds = \lambda \int_0^1 m'(\lambda t)\,dt, \tag{84}$$

where the second equality follows from the change of variables $s = \lambda t$. Using (84),

$$\eta_m(\lambda) = \frac{\lambda m'(\lambda)}{m(\lambda)} = \frac{m'(\lambda)}{\int_0^1 m'(\lambda t)\, dt}. \tag{85}$$

Taking logs and differentiating with respect to λ yields

$$\frac{d}{d\lambda} \log \eta_m(\lambda) = \frac{m''(\lambda)}{m'(\lambda)} - \frac{\int_0^1 t m''(\lambda t)\, dt}{\int_0^1 m'(\lambda t)\, dt}. \tag{86}$$

Next, we can rewrite integrand in the numerator of the second term as

$$tm''(\lambda t) = \frac{\eta_{m'}(\lambda t)}{\lambda} m'(\lambda t).$$

Substituting the above into (86) and multiplying both sides by λ, we obtain

$$\lambda \frac{d}{d\lambda} \log \eta_m(\lambda) = \eta_{m'}(\lambda) - \frac{\int_0^1 \eta_{m'}(\lambda t)\, m'(\lambda t)\, dt}{\int_0^1 m'(\lambda t)\, dt}. \tag{87}$$

The second term on the right-hand side of (87) is a weighted average of $\eta_{m'}(\lambda t)$ over $t \in [0, 1]$ with strictly positive weights proportional to $m'(\lambda t)$. If $\eta_{m'}$ is strictly decreasing, then $\eta_{m'}(\lambda t) > \eta_{m'}(\lambda)$ for all $t \in (0, 1)$, implying this weighted average exceeds $\eta_{m'}(\lambda)$. So $\lambda \frac{d}{d\lambda} \log \eta_m(\lambda) < 0$, which implies $\eta_m'(\lambda) < 0$ for all $\lambda > 0$. □

Proof of Proposition 6

Proof. 1. By Corollary 1, for any $x \in (0, 1)$, we have

$$p(x) = c + \frac{Q_1(\lambda)}{Q_1(\lambda x)}(v - c),$$

which is (22). By Lemma 2 part (vii), $Q_1(\lambda) = -P_0'(\lambda)$ and $\lim_{\lambda \to 0} Q_1(\lambda) = 1$. Therefore, for any fixed $x \in (0, 1)$, we have $\lambda x \to 0$ as $\lambda \to 0$ and hence

$$\lim_{\lambda \to 0} \frac{Q_1(\lambda)}{Q_1(\lambda x)} = 1.$$

Substituting into (22) yields $\lim_{\lambda \to 0} p(x) = v$ for all $x \in (0, 1)$. It follows that F converges to a degenerate distribution at $p = v$.

2. For any $x \in (0, 1)$, $\mu(x) = p(x)/c$. Since $p(x) \to v$, we have $\mu(x) \to v/c = \bar{\mu}$.

3. If F is degenerate at v, then the transaction price distribution H is also

degenerate at $p = v$ in the limit $\lambda \to 0$ because H is the distribution of the lowest price quote a buyer obtains from the degenerate distribution F.

4. By (39) in Lemma 4, the aggregate markup is given by

$$\tilde{\mu}(\lambda) = 1 + \eta_m(\lambda)\,(\bar{\mu} - 1)$$

so it suffices to show that $\eta_m(\lambda) \to 1$ as $\lambda \to 0$. Recall $m(\lambda) = 1 - P_0(\lambda)$ and $\eta_m(\lambda) = \lambda m'(\lambda)/m(\lambda)$. By Lemma 2 parts (iii) and (v), $P_0(\lambda) \to 1$ and $P_0'(\lambda) \to -1$ as $\lambda \to 0$, hence $m(\lambda) \to 0$ and $m'(\lambda) = -P_0'(\lambda) \to 1$. Moreover, by L'Hôpital's rule,

$$\lim_{\lambda \to 0} \frac{m(\lambda)}{\lambda} = \lim_{\lambda \to 0} m'(\lambda) = 1,$$

so $\eta_m(\lambda) = \lambda m'(\lambda)/m(\lambda) \to 1$ as $\lambda \to 0$. Therefore $\tilde{\mu}(\lambda) \to 1 + (\bar{\mu} - 1) = \bar{\mu}$. $\square$

Proof of Lemma 6

Proof. Consider any $x \in (0, 1)$ and define for any $\lambda > 0$ the following:

$$\psi_\lambda(x) \equiv \frac{Q_1(\lambda)}{Q_1(\lambda x)}. \tag{88}$$

By Lemma 2, Q_1 is strictly positive, continuously differentiable, and strictly decreasing on $\mathbb{R}_+$, so $\psi_\lambda(x)$ is well defined. Next, we can write

$$\log \psi_\lambda(x) = \log Q_1(\lambda) - \log Q_1(\lambda x) = \int_{\lambda x}^{\lambda} \frac{Q_1'(t)}{Q_1(t)}\, dt. \tag{89}$$

Recalling the definition of the elasticity η_{Q_1} in (41), we have

$$\log \psi_\lambda(x) = \int_{\lambda x}^{\lambda} \frac{\eta_{Q_1}(t)}{t}\, dt = \int_{x}^{1} \frac{\eta_{Q_1}(\lambda u)}{u}\, du, \tag{90}$$

where the second equality follows from the change of variables $t = \lambda u$.

By Assumption 1, η_{Q_1} is strictly decreasing. Hence, for all $u \in [x, 1]$,

$$\eta_{Q_1}(\lambda) \le \eta_{Q_1}(\lambda u) \le \eta_{Q_1}(\lambda x).$$

Dividing the above by $u > 0$ and integrating over $u \in [x, 1]$ yields

$$\eta_{Q_1}(\lambda) \log\left(\frac{1}{x}\right) \le \log \psi_\lambda(x) \le \eta_{Q_1}(\lambda x) \log\left(\frac{1}{x}\right). \tag{91}$$

Since η_{Q_1} is monotone, the limit $\eta_\infty \equiv \lim_{\lambda\to\infty} \eta_{Q_1}(\lambda) \in [-\infty, 0)$ exists.

Letting $\lambda \to \infty$ in (91) implies that $\log\psi_\lambda(x)$ converges, and therefore the limit $\psi(x) \equiv \lim_{\lambda\to\infty} \psi_\lambda(x)$ exists for all $x \in (0,1)$. If $\eta_\infty = -\infty$, then (91) implies $\log\psi_\lambda(x) \to -\infty$, so $\psi(x) = 0$ for all $x \in (0,1)$. If $\eta_\infty \in (-\infty, 0)$, then (91) yields

$$\log\psi(x) = \eta_\infty \log\left(\frac{1}{x}\right)$$

and thus $\psi(x) = x^{-\eta_\infty}$. Because $-\eta_\infty > 0$, ψ is strictly increasing on $(0,1)$. □

Proof of Proposition 7

Proof. Suppose that P_k is invariant with mean λ and Assumption 1 holds, so that the limit $\psi(x)$ defined by (42) exists for all $x \in (0,1)$. By Lemma 6, either we have $\psi(x) = 0$ for all $x \in (0,1)$ or otherwise ψ is strictly increasing on $(0,1)$.

Starting with Corollary 1, taking $\lambda \to \infty$, and applying (42) yields

$$p_\infty(x) \equiv \lim_{\lambda\to\infty} p(x) = c + \psi(x)(v - c).$$

1. If $\psi(x) = 0$ for all $x \in (0,1)$, then $p(x) \to c$ for all $x \in (0,1)$, hence F converges to a degenerate distribution at $p = c$.

2. If ψ is strictly increasing on $(0,1)$, then p_∞ is strictly increasing and $x = F_\infty(p_\infty)$ for a non-degenerate limit distribution F_∞ with support $[c, v]$ given by (43).

3. For any $x \in (0,1)$, we have $\lim_{\lambda\to\infty} \mu(x) = 1 + \psi(x)\left(\bar{\mu} - 1\right)$, which is (44).

4. Using (6) in Lemma 1, we can write $h(p) = f(p)q(p)/\tilde{q}(\lambda)$, which delivers

$$1 - H(p) = \int_p^v h(t)\,dt = \frac{1}{\tilde{q}(\lambda)} \int_p^v q(t)f(t)\,dt \leq \frac{q(p)}{\tilde{q}(\lambda)}\left(1 - F(p)\right)$$

where the inequality uses the fact that $q(\cdot)$ is weakly decreasing in p. Next, by Proposition 4, we have $q(p;\lambda) = Q_1\left(\lambda F(p;\lambda)\right)$ and $\tilde{q}(\lambda) = m(\lambda)/\lambda$, and therefore

$$1 - H(p;\lambda) \leq \frac{\lambda Q_1\left(\lambda F(p;\lambda)\right)}{m(\lambda)}\left(1 - F(p;\lambda)\right) = \frac{P_1\left(\lambda F(p;\lambda)\right)}{m(\lambda)F(p;\lambda)}\left(1 - F(p;\lambda)\right).$$

It follows from the equilibrium pricing equations (19) and (20) that $\lambda F(p;\lambda) \to \infty$ as $\lambda \to \infty$ because $Q_1(\lambda) \to 0$ as $\lambda \to \infty$. By Lemma 11 below, $P_1(t) \to 0$ as $t \to \infty$, and by Lemma 2 part (iv), we have $m(\lambda) = 1-P_0(\lambda) \to 1$. Therefore, $1-H(p;\lambda) \to 0$ for every $p > c$, which implies that H converges to a degenerate distribution at $p = c$.

5. By (39), we have $\tilde{\mu}(\lambda) = 1+\eta_m(\lambda)\,(\bar{\mu}-1)$, so it suffices to show that $\eta_m(\lambda) \to 0$ as $\lambda \to \infty$. Under invariance, we have $-P_0'(\lambda) = Q_1(\lambda) = P_1(\lambda)/\lambda$, and therefore

$$\eta_m(\lambda) = \frac{-\lambda P_0'(\lambda)}{1-P_0(\lambda)} = \frac{P_1(\lambda)}{1-P_0(\lambda)}.$$

Now, $P_0(\lambda) \to 0$ and $P_1(\lambda) \to 0$ by Lemma 11, so $\eta_m(\lambda) \to 0$ and $\tilde{\mu}(\lambda) \to 1$ in the limit as $\lambda \to \infty$. □

Lemma 11. *If P_k is invariant, then*

$$\lim_{\lambda\to\infty} P_1(\lambda) = 0.$$

Proof. Setting $k = 1$ in expression (53) gives

$$P_1(\lambda) = \int_0^\infty \lambda\tau e^{-\lambda\tau}\, d\Phi(\tau).$$

For each $\tau \geq 0$, we have $\lambda\tau e^{-\lambda\tau} \to 0$ as $\lambda \to \infty$. Moreover, letting $y = \lambda\tau$, the bound $ye^{-y} \leq 1/e$ for all $y \geq 0$ implies $0 \leq \lambda\tau e^{-\lambda\tau} \leq 1/e$ for all $\lambda > 0$ and $\tau \geq 0$. Since $1/e$ is Φ-integrable, the dominated convergence theorem yields $P_1(\lambda) \to 0$. □

A.3 Proofs for Section 5

Proof of Lemma 7

Proof. Standard properties of a probability generating function $\mathbb{G}$ imply

$$\sigma^2(\lambda) = \mathbb{G}''(1) + \lambda - \lambda^2$$

where λ is the mean. Therefore, we have

$$cv^2(\lambda) = \frac{\sigma^2(\lambda)}{\lambda^2} = \frac{\mathbb{G}''(1)}{\lambda^2} + \frac{1}{\lambda} - 1. \tag{92}$$

If P_k is invariant, $\mathbb{G}(z) = P_0(\lambda(1-z))$ by (16) in Definition 1. Differentiating yields

$$\mathbb{G}'(z) = -\lambda P_0'\big(\lambda(1-z)\big), \qquad \mathbb{G}''(z) = \lambda^2 P_0''\big(\lambda(1-z)\big).$$

Evaluating at $z = 1$ and substituting into (92), and then using the fact that $Q_1(\lambda) = -P_0'(\lambda)$ by Lemma 2 and thus $Q_1'(\lambda) = -P_0''(\lambda)$, gives us the desired result. $\square$

Proof of Proposition 9

Proof. Let $\tilde{\mu}(\lambda; P)$ and $\tilde{\mu}(\lambda; G)$ denote the aggregate markup under the Poisson and geometric search technologies, respectively. By (34) and (109), we have

$$\tilde{\mu}(\lambda; G) - \tilde{\mu}(\lambda; P) = (\bar{\mu} - 1)\left[\frac{1}{1+\lambda} - \frac{\lambda e^{-\lambda}}{1 - e^{-\lambda}}\right].$$

Since $\bar{\mu} - 1 > 0$, the sign of $\tilde{\mu}(\lambda; G) - \tilde{\mu}(\lambda; P)$ is the sign of

$$\Delta(\lambda) \equiv \frac{1}{1+\lambda} - \frac{\lambda e^{-\lambda}}{1 - e^{-\lambda}} = \frac{1 - e^{-\lambda}\left(1 + \lambda + \lambda^2\right)}{(1+\lambda)\left(1 - e^{-\lambda}\right)}.$$

The denominator is strictly positive for all $\lambda > 0$, so $\Delta(\lambda)$ has the same sign as

$$g(\lambda) \equiv e^{\lambda} - \left(1 + \lambda + \lambda^2\right).$$

Note that $g(0) = 0$. For small $\lambda > 0$ we have $e^{\lambda} = 1 + \lambda + \lambda^2/2 + o\left(\lambda^2\right)$, so $g(\lambda) < 0$ for λ close to 0. Also $g(\lambda) \to \infty$ as $\lambda \to \infty$. Moreover, $g''(\lambda) = e^{\lambda} - 2$ is strictly increasing in λ, implying g' is strictly decreasing on $(0, \ln 2)$ and strictly increasing on $(\ln 2, \infty)$. Since $g'(0) = 0$ and $g''(0) = -1$, we have $g'(\lambda) < 0$ for λ small, while $g'(\lambda) \to \infty$ as $\lambda \to \infty$, so there exists a unique $\lambda_0 > 0$ with $g'(\lambda_0) = 0$. Therefore g is strictly decreasing on $(0, \lambda_0)$ and strictly increasing on (λ_0, ∞). Because $g(0) = 0$ and g is negative for λ just above 0 and tends to $+\infty$, it follows that there exists a unique $\lambda_c^* > 0$ such that $g(\lambda_c^*) = 0$. Finally, since $\tilde{\mu}(\lambda; G) - \tilde{\mu}(\lambda; P)$ has the same sign as $g(\lambda)$, it is negative for $\lambda < \lambda_c^*$ and positive for $\lambda > \lambda_c^*$. $\square$

Proof of Proposition 10

Proof. By Lemma 4, we have the following expression for the aggregate markup:

$$\tilde{\mu}(\lambda; r) = 1 + \eta_m(\lambda; r)\left(\bar{\mu} - 1\right), \tag{93}$$

Therefore, we have that $\tilde{\mu}(\lambda; r)$ is strictly increasing (decreasing) in r if and only if $\eta_m(\lambda; r)$ is strictly increasing (decreasing) in r since $\bar{\mu} > 1$.

For the negative binomial family, $\eta_m(\lambda; r)$ is given by (115). The proof of Proposition 2 in Mangin and Zhang (2026) shows that, letting $\lambda_d^* \approx 1.59$ solve $e^{\lambda} = 2/(2-\lambda)$, the function $\eta_m(\lambda; r)$ is strictly decreasing in competitive dispersion $1/r$ when $\lambda < \lambda_d^*$, while for $\lambda > \lambda_d^*$ it is increasing in $1/r$ for low dispersion and decreasing in $1/r$ for high dispersion, with a unique cut-off $r^*(\lambda) \in (0, \infty)$ at which the change occurs. Therefore, the same comparative statics hold for the aggregate markup $\tilde{\mu}(\lambda; r)$. □

Proof of Proposition 11

Proof. Under the Poisson search technology, equilibrium price dispersion is

$$cv_H^2(\lambda; P) = \left(\frac{e^{-\lambda}}{\hat{\eta}_m(\lambda)^2} - 1\right)\left(\frac{(v-c)\,\hat{\eta}_m(\lambda)}{c + (v-c)\,\hat{\eta}_m(\lambda)}\right)^2, \tag{94}$$

where $\hat{\eta}_m(\lambda) \equiv \frac{\lambda e^{-\lambda}}{1-e^{-\lambda}}$. Rearranging (94) yields the equivalent form

$$cv_H^2(\lambda; P) = \left(e^{-\lambda} - \hat{\eta}_m(\lambda)^2\right)\left(\frac{v-c}{c + (v-c)\,\hat{\eta}_m(\lambda)}\right)^2. \tag{95}$$

Under the geometric search technology, equilibrium price dispersion is

$$cv_H^2(\lambda; G) = \frac{\lambda^2}{3\,(1+\lambda)}\left(\frac{v-c}{v+c\lambda}\right)^2. \tag{96}$$

Therefore, the inequality $cv_H^2(\lambda; G) > cv_H^2(\lambda; P)$ is equivalent to

$$\frac{\lambda^2}{3\,(1+\lambda)}\frac{1}{(v+c\lambda)^2} > \frac{e^{-\lambda} - \hat{\eta}_m(\lambda)^2}{(c + (v-c)\,\hat{\eta}_m(\lambda))^2}. \tag{97}$$

Using $\bar{\mu} \equiv v/c$, we have $v + c\lambda = c\,(\bar{\mu} + \lambda)$ and $c + (v-c)\,\hat{\eta}_m(\lambda) = c\,(1 + (\bar{\mu} - 1)\,\hat{\eta}_m(\lambda))$. Substituting into (97) yields

$$\frac{1 + (\bar{\mu} - 1)\;\hat{\eta}_m(\lambda)}{\bar{\mu} + \lambda} > \frac{\sqrt{3\,(1+\lambda)\,(e^{-\lambda} - \;\hat{\eta}_m(\lambda)^2)}}{\lambda}. \tag{98}$$

The inequality $cv_H^2(\lambda; G) > cv_H^2(\lambda; P)$ holds if and only if the above inequality holds.

It is straightforward to verify numerically that inequality (98) holds and thus $cv_H^2(\lambda; G) > cv_H^2(\lambda; P)$ for *any* $\bar{\mu} \geq 1$ under the sufficient condition $\lambda \leq 2$. □

Proof of Proposition 12

Proof. For the negative binomial family, the closed-form expression for equilibrium price dispersion is given by equation (116) and when $c = 0$ this reduces to

$$cv_H^2(\lambda; r) = \frac{r}{r+2} \frac{(1 - a(r))(1 - b(r))}{\lambda^2 b(r)} - 1, \tag{99}$$

where we define $a(r) \equiv \left(\frac{r}{r+\lambda}\right)^r$ and $b(r) \equiv \left(\frac{r}{r+\lambda}\right)^{r+2}$.

In the Poisson limit $r \to \infty$, we have $a(r) \to e^{-\lambda}$ and $b(r) \to e^{-\lambda}$. The sign of the local effect of competitive dispersion $\delta \equiv 1/r$ is the opposite of the sign of $\frac{\partial}{\partial r} cv_H^2(\lambda; r)$ as $r \to \infty$. Differentiating (99) and taking the limit as $r \to \infty$ yields

$$\lim_{r \to \infty} \frac{\partial}{\partial \delta} cv_H^2(\lambda; r) = \frac{\Delta_P(\lambda)}{2\lambda^2} \tag{100}$$

where

$$\Delta_P(\lambda) \equiv (2 - \lambda)\left(1 - e^{-\lambda}\right)\left(\lambda + 2 + (\lambda - 2)\, e^{\lambda}\right). \tag{101}$$

The denominator in (100) is strictly positive for all $\lambda > 0$. Moreover, $1 - e^{-\lambda} > 0$ for all $\lambda > 0$ and it can be verified that $\lambda + 2 + (\lambda - 2)\, e^{\lambda} > 0$ for all $\lambda > 0$. So, the sign of the local effect of competitive dispersion near the Poisson is the sign of $2 - \lambda$.

It follows that price dispersion $cv_H^2(\lambda; r)$ is strictly increasing in dispersion $1/r$ in a neighborhood of the Poisson if $\lambda < 2$ and strictly decreasing in $1/r$ if $\lambda > 2$. □

A.4 Proofs for Section 6

Proof of Proposition 14

Proof. 1. We know from Lemma 7 that, for any invariant search technology,

$$cv^2(\lambda) = \frac{1}{\lambda} + P_0''(0) - 1 \tag{102}$$

because $P_0''(0) = -Q_1'(0)$. Since P_k is invariant, then for all $x \geq 0$, we have

$$P_0(x) = \int_0^\infty e^{-x\tau}\, d\Phi(\tau)$$

for some distribution Φ with mean one. Differentiating twice yields

$$P_0''(0) = \int_0^\infty \tau^2 \, d\Phi(\tau).$$

By Jensen's inequality, we know that

$$\int_0^\infty \tau^2 \, d\Phi(\tau) \;\geq\; \left(\int_0^\infty \tau \, d\Phi(\tau)\right)^2 = 1,$$

with equality if and only if Φ is degenerate at $\tau = 1$, i.e. if and only if P_k is Poisson. Therefore $P_0''(0) \geq 1$ and (102) implies $cv^2(\lambda) \geq 1/\lambda$, which is the Poisson $cv^2(\lambda)$.

2. Welfare $W(\lambda; P_k)$ depends on the search technology only through the term $m(\lambda; P_k) = 1 - P_0(\lambda)$, so it suffices to consider only $P_0(\lambda)$. By Lemma 9, $P_0(\lambda)$ has a Laplace transform representation, $P_0(\lambda) = \mathbb{E}_\Phi[e^{-\lambda\tau}]$. Since $e^{-\lambda\tau}$ is convex in τ, Jensen's inequality implies that $P_0(\lambda)$ is strictly increasing in a mean-preserving spread of Φ. Therefore, $P_0(\lambda)$ is minimized if Φ is degenerate, i.e. if and only if P_k is Poisson. So $W(\lambda; P_k)$ and $m(\lambda; P_k)$ are maximized if and only if P_k is Poisson.

3. Part 2 implies that, for any given λ, the planner would choose P_k to be Poisson from within the invariant class, so the efficient competitive dispersion is Poisson. □

A.5 Proofs for Section 7

Proof of Proposition 15

Proof. Consider a distribution of competitive intensity Φ that satisfies our assumptions. It generates the unique search technology P_k described by (53) in Definition 2, which is a mixed Poisson search technology with mixing distribution Φ. This search technology is invariant by Definition 1 because its probability generating function is

$$\mathbb{G}(z) = \sum_{k=0}^{\infty} P_k(\lambda) z^k = \int_0^\infty \sum_{k=0}^{\infty} \frac{(\lambda\tau z)^k \, e^{-\lambda\tau}}{k!} d\Phi(\tau) = \int_0^\infty e^{-\lambda\tau(1-z)} d\Phi(\tau) = P_0\left(\lambda\left(1-z\right)\right).$$

Now consider any invariant search technology P_k. By Lemma 9, which was proven in Cai et al. (2025), we have the following Laplace transform representation:

$$P_0(\lambda) = \int_0^\infty e^{-\lambda\tau} d\Phi(\tau) \tag{103}$$

for some distribution Φ with mean $\mathbb{E}_\Phi[\tau] = 1$. By Corollary 6 in Mangin (2026), it follows that P_k is a mixed Poisson distribution with mixing distribution Φ. So Φ is the unique distribution of competitive intensity that generates P_k by Definition 2. $\square$

Proof of Corollary 8

Proof. 1. For a buyer with competitive intensity $\tau > 0$, the distribution $P_k^\tau(\lambda\tau)$ of the number of sellers they meet is Poisson with mean $\lambda\tau$. For any $N \geq 0$,

$$\Pr\left(K \leq N|\tau\right) = e^{-\lambda\tau}\sum_{k=0}^{N}\frac{\left(\lambda\tau\right)^k}{k!}.$$

Differentiating with respect to τ gives

$$\frac{\partial}{\partial\tau}\Pr\left(K \leq N|\tau\right) = -\lambda e^{-\lambda\tau}\frac{\left(\lambda\tau\right)^N}{N!} < 0.$$

Thus the cdf of $K|\tau$ is decreasing in τ, so $P_k^\tau\left(\lambda\tau_1\right) \preccurlyeq_{FOSD} P_k^\tau\left(\lambda\tau_2\right)$ for any $\tau_1 < \tau_2$.

2. By Proposition 16, the conditional cdf of transaction prices satisfies (54). For each p, expression (54) is increasing in τ, thus $H(\cdot|\tau_2) \preccurlyeq_{FOSD} H(\cdot|\tau_1)$ for any $\tau_2 > \tau_1$.

3. It follows from part 2 that $\tilde{p}(\lambda|\tau) \equiv \mathbb{E}_{H(\cdot|\tau)}[p]$ is decreasing in τ.

4. The lower bound of the support of F is $\underline{p} = c + Q_1(\lambda)(v - c)$. For any $p > \underline{p}$, we have $F(p) > 0$ and therefore $e^{-\lambda\tau F(p)} \to 0$ and $e^{-\lambda\tau} \to 0$ as $\tau \to \infty$. So $\lim_{\tau\to\infty} H(p|\tau) = 1$ for all $p > \underline{p}$. It follows that, as $\tau \to \infty$, the distribution $H(\cdot|\tau)$ converges to a degenerate distribution at $\underline{p}$. Consequently, $cv_H^2(\lambda|\tau) \to 0$ as $\tau \to \infty$.

5. From expression (54) in Proposition 16, for any $\tau > 0$ in the support of Φ,

$$H(p|\tau) = \frac{1 - e^{-\lambda\tau F(p)}}{1 - e^{-\lambda\tau}}.$$

Fix any $p \in [\underline{p}, v]$. As $\tau \to 0$, both the numerator and denominator converge to zero. Applying L'Hôpital's rule gives

$$\lim_{\tau\to 0} H(p|\tau) = \lim_{\tau\to 0}\frac{F(p)e^{-\lambda\tau F(p)}}{e^{-\lambda\tau}} = F(p).$$

So $H(\cdot|\tau) \Rightarrow F(\cdot)$ as $\tau \to 0$, hence

$$\mathbb{E}_{H(\cdot|\tau)}[p] \to \mathbb{E}_F[p] \qquad \text{and} \qquad \mathbb{E}_{H(\cdot|\tau)}[p^2] \to \mathbb{E}_F[p^2].$$

Hence we have $\sigma_H^2(\lambda|\tau) \to \sigma_F^2(\lambda)$ and therefore $cv_H^2(\lambda|\tau) \to cv_F^2(\lambda)$ as $\tau \to 0$. $\square$

A.6 Proofs for Section 8

Proof of Proposition 17

Proof. Define match profit $p \equiv y - w$. A worker maximizes w by minimizing p, so the labor market is isomorphic to Section 2 with $c = 0$, $v = y - z$, and $\lambda = \theta$.

Let F denote the equilibrium cdf of posted profits $p = y - w$. By Proposition 1, there exists a unique symmetric mixed-strategy equilibrium, F has no mass points, and its support is an interval $\left[\underline{p}, v\right]$ where $\underline{p} = c + Q_1(\theta)\,(v - c) = Q_1(\theta)\,(y - z)$. Since $w = y - p$, the support of the distribution of posted wages F_w is $[z, \bar{w}]$ where $\bar{w} = y - Q_1(\theta)\,(y - z)$. The relationship between F_w and F is as follows:

$$F_w(w) = \Pr\left(W \leq w\right) = \Pr\left(P \geq y - w\right) = 1 - F\left(y - w\right) = 1 - F(p). \tag{104}$$

If P_k is invariant, Proposition 4 applied to the profit problem (with $c = 0$, $v = y - z$, $\lambda = \theta$) yields $F(p) = \frac{1}{\theta}\left(Q_1\right)^{-1}\left(Q_1(\theta)\frac{y-z}{p}\right)$. Substituting in $p = y - w$ yields (56).

Next, under invariance, Proposition 4 implies that any posted profit p satisfies $p = \frac{Q_1(\theta)}{q_p(p)}\,(y - z)$ and $q_p(p) = Q_1(\theta F(p))$. Using $p = y - w$ and $F(p) = 1 - F_w(w)$, define $q_w(w) \equiv q_p\left(y - w\right)$ to obtain $q_w(w) = Q_1(\theta\left(1 - F_w(w)\right))$, which is (58). Finally, rewriting $y - w = \frac{Q_1(\theta)}{q_w(w)}\,(y - z)$ gives $w = z + \left(1 - \frac{Q_1(\theta)}{q_w(w)}\right)(y - z)$, which is (57). $\square$

Proof of Lemma 8

Proof. Conditional on receiving $k \geq 1$ offers, the accepted wage is the maximum of k i.i.d. draws from F_w. Conditioning on $K \geq 1$ therefore gives us

$$H_w(w) = \frac{\sum_{k=1}^{\infty} P_k(\theta) F_w(w)^k}{1 - P_0(\theta)}.$$

Applying Definition 1 and subtracting the $k = 0$ term $P_0(\theta)$ yields (59).

For wage dispersion, note that $p \equiv y - w$ is a linear transformation, so $\sigma^2_{H_w}(\theta) = \sigma^2_H(\theta)$, where H is the accepted profit distribution. Let $\tilde{p}(\theta) \equiv \mathbb{E}_H\left[p\right]$. Since $p = y - w$, we have $\tilde{p}(\theta) = y - \tilde{w}(\theta)$ and $\mathbb{E}_F\left[p\right] = \mathbb{E}_F\left[y - w\right]$. By Proposition 3, in the $c = 0$ case we have $cv^2_H(\theta) = \mathbb{E}_F\left[p\right]/\tilde{p}(\theta) - 1$ as in (12). Therefore,

$$cv^2_{H_w}(\theta) = \frac{\sigma^2_{H_w}(\theta)}{\tilde{w}(\theta)^2} = \frac{\sigma^2_H(\theta)}{\tilde{w}(\theta)^2} = cv^2_H(\theta)\left(\frac{\tilde{p}(\theta)}{\tilde{w}(\theta)}\right)^2,$$

which is exactly (60) after substituting $\tilde{p}(\theta) = y - \tilde{w}(\theta)$ and $\mathbb{E}_F\left[p\right] = \mathbb{E}_{F_w}\left[y - w\right]$. $\square$

Proof of Proposition 19

Proof. For parts 2 and 3, Proposition 18 implies that the average wage is given by (61), which is increasing in competitive dispersion if and only if $\alpha(\theta)$ is increasing in competitive dispersion. By Corollary 3, the aggregate markup is increasing in competitive dispersion if and only if the function α is decreasing in competitive dispersion. Therefore, parts 2 and 3 follow from Proposition 10 translated to the labor market and with the directions reversed. Part 1 follows directly from Lemma 12 below. $\square$

Lemma 12. *If P_k is negative binomial family with mean θ and competitive dispersion $1/r$, then $P_0(\theta; r)$ is strictly increasing in competitive dispersion $1/r$.*

Proof. For any given $\theta > 0$, we can define

$$g(r) \equiv \log P_0(\theta; r) = r \log\left(\frac{r}{r+\theta}\right).$$

It suffices to determine the sign of $g'(r)$. Differentiating the above yields

$$g'(r) = \log\left(\frac{r}{r+\theta}\right) + r\left(\frac{1}{r} - \frac{1}{r+\theta}\right) = \log\left(\frac{r}{r+\theta}\right) + \frac{\theta}{r+\theta}.$$

Letting $x \equiv \theta/(r+\theta) \in (0,1)$, we can write $g'(r) = \log(1-x)+x$. Since $\log(1-x) < -x$ for all $x \in (0,1)$, we have $g'(r) < 0$ for all $r > 0$ and $\theta > 0$. Therefore $P_0(\theta; r)$ is strictly decreasing in r and strictly increasing in competitive dispersion $1/r$. $\square$

Proof of Proposition 20

Proof. Assume $z = 0$ and let $\delta \equiv 1/r$. From Lemma 8, wage dispersion is

$$cv^2_{H_w}(\theta; r) = \left(\frac{\mathbb{E}_F[p]}{\tilde{p}(\theta; r)} - 1\right)\left(\frac{\tilde{p}(\theta; r)}{\tilde{w}(\theta; r)}\right)^2 \tag{105}$$

where $p \equiv y - w$, $\tilde{p}(\theta; r) = \mathbb{E}_H[p] = y - \tilde{w}(\theta; r)$, and $\mathbb{E}_F[p] = \mathbb{E}_{F_w}[y - w]$.

For the negative binomial family, the local effect of δ on the first term in (105) at the Poisson limit ($r \to \infty$) is given by the proof of Proposition 12 as follows:

$$\lim_{r\to\infty} \frac{\partial}{\partial \delta}\left(\frac{\mathbb{E}_F[p]}{\tilde{p}(\theta; r)} - 1\right) = \frac{\Delta_P(\theta)}{2\theta^2}.$$

Now consider the second term on the right-hand side of (105). Under invariance and at $z = 0$, Proposition 18 implies that

$$\frac{\tilde{p}(\theta; r)}{\tilde{w}(\theta; r)} = \frac{\eta_m(\theta; r)}{1 - \eta_m(\theta; r)}. \tag{106}$$

For the negative binomial family, we have

$$\eta_m(\theta; r) = \frac{\theta \left(\frac{r}{r+\theta}\right)^{r+1}}{1 - \left(\frac{r}{r+\theta}\right)^{r}}.$$

A first-order expansion around $\delta = 0$ (or $r \to \infty$) yields

$$\lim_{r \to \infty} \frac{\partial}{\partial \delta} \eta_m(\theta; r) = \frac{\theta^3 e^\theta}{2\left(e^\theta - 1\right)^2} - \frac{\theta^2}{e^\theta - 1}.$$

Differentiating $cv^2_{H_w}(\theta; r)$ with respect to δ at $\delta = 0$ (or $r \to \infty$) by combining the previous two steps via the product rule and simplifying yields

$$\lim_{r \to \infty} \frac{\partial}{\partial \delta} cv^2_{H_w}(\theta; r) = \frac{\Delta_{LM}(\theta)}{2\left(e^\theta - \theta - 1\right)^3 e^\theta},$$

for some function $\Delta_{LM}(\theta)$. The denominator is strictly positive for all $\theta > 0$ because $e^\theta > 1 + \theta$. Therefore, the sign of $\frac{\partial}{\partial \delta} cv^2_{H_w}(\theta; r)$ at $\delta = 0$ is the sign of $\Delta_{LM}(\theta)$. It is straightforward to verify numerically that $\Delta_{LM}(\theta)$ has a unique root $\theta^*_z \approx 0.91$, with $\Delta_{LM}(\theta) < 0$ for $\theta < \theta^*_z$ and $\Delta_{LM}(\theta) > 0$ for $\theta > \theta^*_z$. Therefore, in a neighborhood of $\delta = 0$ (Poisson), wage dispersion is strictly decreasing in competitive dispersion $\delta = 1/r$ if $\theta < \theta^*_z$ and strictly increasing in competitive dispersion if $\theta > \theta^*_z$. □

Appendix B: Examples

Example: Geometric search technology

For the geometric search technology with mean λ, we have

$$P_k(\lambda) = \frac{1}{1+\lambda}\left(\frac{\lambda}{1+\lambda}\right)^k.$$

It is invariant by Definition 1 because its probability generating function is

$$\mathbb{G}(z) = \sum_{k=0}^{\infty} P_k(\lambda) z^k = \frac{1}{1+\lambda(1-z)} = P_0(\lambda(1-z)).$$

We have $Q_1(\lambda) = P_1(\lambda)/\lambda = 1/(1+\lambda)^2$ and therefore Proposition 4 implies

$$q(p) = \frac{1}{(1+\lambda F(p))^2}.$$

Given that $Q_1(\lambda) = 1/(1+\lambda)^2$, any posted price p satisfies

$$p = c + \left(\frac{1+\lambda F(p)}{1+\lambda}\right)^2 (v-c).$$

Rearranging, the distribution of posted prices is

$$F(p) = 1 - \frac{1+\lambda}{\lambda}\left[1 - \left(\frac{v-c}{p-c}\right)^{-1/2}\right]. \tag{107}$$

Applying Corollary 4, the equilibrium markup by quantile is

$$\mu(x) = 1 + \left(\frac{1+\lambda x}{1+\lambda}\right)^2 (\bar{\mu} - 1). \tag{108}$$

By Corollary 2, the distribution of transaction prices is

$$H(p) = \frac{(1+\lambda)F(p)}{1+\lambda F(p)}.$$

By Proposition 2, the aggregate price is

$$\tilde{p}(\lambda) = c + (1-\alpha(\lambda))(v-c) \tag{109}$$

where

$$1-\alpha(\lambda)=\frac{1}{1+\lambda}. \tag{110}$$

By Proposition 3, equilibrium price dispersion is given by

$$cv_H^2(\lambda)=\frac{\lambda^2}{3(1+\lambda)}\left(\frac{v-c}{v+c\lambda}\right)^2. \tag{111}$$

Example: Negative binomial family of search technologies

For the negative binomial family with mean λ and parameter $r>0$, we have

$$P_k(\lambda;r)=\binom{k+r-1}{k}\left(\frac{r}{r+\lambda}\right)^r\left(\frac{\lambda}{r+\lambda}\right)^k.$$

It is invariant by Definition 1 because its probability generating function is

$$\mathbb{G}(z)=\sum_{k=0}^{\infty}P_k(\lambda)z^k=\left(\frac{r}{r+\lambda(1-z)}\right)^r=P_0(\lambda(1-z);r).$$

To derive the equilibrium, Proposition 4 and $\lambda Q_k(\lambda)=kP_k(\lambda)$ give

$$q(p)=\left(\frac{r}{r+\lambda F(p)}\right)^{r+1}.$$

Given that $Q_1(\lambda)=\left(\frac{r}{r+\lambda}\right)^{r+1}$, any posted price p satisfies

$$p=c+\left(\frac{r+\lambda F(p)}{r+\lambda}\right)^{r+1}(v-c). \tag{112}$$

Rearranging (112), the distribution of posted prices is

$$F(p)=1-\frac{r+\lambda}{\lambda}\left[1-\left(\frac{v-c}{p-c}\right)^{-\frac{1}{r+1}}\right].$$

Applying Corollary 4, the equilibrium markup by quantile is

$$\mu(x)=1+\left(\frac{r+\lambda x}{r+\lambda}\right)^{r+1}(\bar{\mu}-1). \tag{113}$$

Applying Corollary 2, the distribution of transaction prices is

$$H(p) = \frac{1 - \left(\frac{r}{r+\lambda F(p)}\right)^r}{1 - \left(\frac{r}{r+\lambda}\right)^r}.$$

By Lemma 4, the aggregate price is

$$\tilde{p}(\lambda) = c + \eta_m(\lambda; r)(v - c) \tag{114}$$

where

$$\eta_m(\lambda; r) = \frac{\lambda \left(\frac{r}{r+\lambda}\right)^{r+1}}{1 - \left(\frac{r}{r+\lambda}\right)^r}. \tag{115}$$

By Proposition 3, equilibrium price dispersion is

$$cv_H^2(\lambda; r) = \left[\frac{r}{r+2} \frac{\left(1 - \left(\frac{r}{r+\lambda}\right)^r\right)\left(1 - \left(\frac{r}{r+\lambda}\right)^{r+2}\right)}{\lambda^2 \left(\frac{r}{r+\lambda}\right)^{r+2}} - 1\right] \left(\frac{\tilde{p}(\lambda) - c}{\tilde{p}(\lambda)}\right)^2. \tag{116}$$

Example: Non-invariant search technology

Consider the following (non-invariant) search technology:

$$P_0(\lambda) = \frac{1}{1+\lambda} \qquad \text{and} \qquad P_k(\lambda) = \frac{\lambda}{1+\lambda} \frac{e^{-\lambda}\lambda^{k-1}}{(k-1)!} \quad \text{for } k \geq 1.$$

Equivalently, with probability $1/(1+\lambda)$ a buyer meets no sellers, and conditional on meeting at least one seller the buyer meets $K = 1 + N$ sellers where $N \sim \text{Poisson}(\lambda)$.

This distribution has mean λ and its probability generating function is

$$\mathbb{G}(z) = \sum_{k=0}^{\infty} P_k(\lambda) z^k = \frac{1}{1+\lambda} + \frac{\lambda}{1+\lambda} z e^{\lambda(z-1)}.$$

This is *not* invariant by Definition 1 because $\mathbb{G}(z) \neq P_0(\lambda(1-z))$ for $z \in (0,1)$.

For this example, $m(\lambda) = 1 - P_0(\lambda) = \lambda/(1+\lambda)$ and $Q_1(\lambda) = P_1(\lambda)/\lambda = e^{-\lambda}/(1+\lambda)$. The quantity demanded at posted price p is given by

$$q(p) = \sum_{k=1}^{\infty} Q_k(\lambda)(1 - F(p))^{k-1} = \frac{1 + \lambda(1 - F(p))}{1+\lambda} e^{-\lambda F(p)}.$$

Applying Proposition 1, any posted price p satisfies

$$p = c + \frac{e^{-\lambda(1-F(p))}}{1+\lambda\,(1-F(p))}(v-c). \tag{117}$$

The equilibrium markup by quantile is

$$\mu(x) = 1 + \frac{e^{-\lambda(1-x)}}{1+\lambda\,(1-x)}\,(\bar{\mu}-1)\,. \tag{118}$$

By Lemma 1, the distribution of transaction prices is

$$H(p) = 1 - (1-F(p))\,e^{-\lambda F(p)}.$$

By Proposition 2, the proportion of monopoly sales is

$$1-\alpha(\lambda) = \frac{Q_1(\lambda)}{\tilde{q}(\lambda)} = e^{-\lambda}$$

and therefore the aggregate price is

$$\tilde{p}(\lambda) = c + e^{-\lambda}(v-c). \tag{119}$$

By Proposition 13, we do not have constrained efficiency. The matching elasticity is

$$\eta_m(\lambda) = \frac{1}{1+\lambda}, \tag{120}$$

which is not equal to the proportion of monopoly sales, $1-\alpha(\lambda) = e^{-\lambda}$.

References

J. Albrecht, G. Menzio, and S. Vroman. Vertical differentiation in frictional product markets. *Journal of Political Economy Macroeconomics*, 1(3):586–632, 2023.

K. Burdett and K. L. Judd. Equilibrium price dispersion. *Econometrica*, 51(4): 955–969, 1983.

K. Burdett, A. Trejos, and R. Wright. A new suggestion for simplifying the theory of money. *Journal of Economic Theory*, 172:423–450, 2017.

G. Butters. Equilibrium Distributions of Sales and Advertising Prices. *Review of Economic Studies*, 44:465–491, 1977.

X. Cai, P. Gautier, and R. Wolthoff. Spatial search. *Journal of Economic Theory*, 224:105976, 2025.

A. Chernoff, A. Head, and B. Lapham. Consumer search, productivity heterogeneity, prices, markups, and pass-through: Theory and estimation. Manuscript, 2024.

A. Chernoff, A. Head, and B. Lapham. Markups, pass-through, and firm heterogeneity with sequentially mixed search. Manuscript, 2025.

J. Eeckhout and P. Kircher. Sorting versus screening: Search frictions and competing mechanisms. *Journal of Economic Theory*, 145(4):1354–1385, 2010.

A. Head and A. Kumar. Price dispersion, inflation, and welfare. *International Economic Review*, 46(2):533–572, 2005.

A. Head, A. Kumar, and B. Lapham. Market power, price adjustment, and inflation. *International Economic Review*, 51(1):73–98, 2010.

A. Head, L. Q. Liu, G. Menzio, and R. Wright. Sticky prices: A new monetarist approach. *Journal of the European Economic Association*, 10(5):939–973, 2012.

L. Herrenbrueck. An open-economy model with money, endogenous search, and heterogeneous firms. *Economic Inquiry*, 55(4):1648–1670, 2017.

A. J. Hosios. On the Efficiency of Matching and Related Models of Search and Unemployment. *Review of Economic Studies*, 57(2):279–298, 1990.

G. Kaplan and G. Menzio. Shopping externalities and self-fulfilling unemployment fluctuations. *Journal of Political Economy*, 124(3):771–825, 2016.

G. Kaplan, G. Menzio, L. Rudanko, and N. Trachter. Relative price dispersion: Evidence and theory. *American Economic Journal: Microeconomics*, 11(3):68–124, 2019.

B. Lester, L. Visschers, and R. Wolthoff. Meeting technologies and optimal trading mechanisms in competitive search markets. *Journal of Economic Theory*, 155:1–15, 2015.

B. Lester, A. Shourideh, V. Venkateswaran, and A. Zetlin-Jones. Screening and adverse selection in frictional markets. *Journal of Political Economy*, 127(1):338–377, 2019.

S. Mangin. Extreme value theory with heterogeneous agents. *Econometrica (forthcoming)*, 2026.

S. Mangin and G. Menzio. Private and social learning in frictional product markets. *Working paper*, 2024.

S. Mangin and Y. Zhang. Optimal competitive dispersion. Working paper, 2026.

G. Menzio. Optimal product design: Implications for competition and growth under declining search frictions. *Econometrica*, 91(2):605–639, 2023.

G. Menzio. Search theory of imperfect competition with decreasing returns to scale. *Journal of Economic Theory*, 218:105827, 2024a.

G. Menzio. Markups: A search-theoretic perspective. *NBER Working Paper 32888*, 2024b.

G. Menzio and N. Trachter. Equilibrium price dispersion across and within stores. *Review of Economic Dynamics*, 28:205–220, 2018.

D. T. Mortensen. A comment on "price dispersion, inflation, and welfare" by A. Head and A. Kumar. *International Economic Review*, 46(2):573–578, 2005.

L. Nord. Shopping, demand composition, and equilibrium prices. *Working paper*, 2023.

K. Pytka. Shopping frictions with household heterogeneity: Theory and empirics. *Manuscript*, 2024.

K. Sangani. Markups across the income distribution: Measurement and implications. Manuscript, 2025.

S. Shi. Sequentially mixed search and equilibrium price dispersion. *Journal of Economic Theory*, 213:105735, 2023.

H. R. Varian. A model of sales. *American Economic Review*, 70(4):651–659, 1980.