Consumer Choice and the Cost of Inflation*

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March 16, 2023

Abstract

Is inflation more or less costly in economies where consumers have a greater degree of informed choice about their purchases? To answer this question, we develop a search-theoretic model of monetary exchange in which the *degree of informed choice* about purchases can vary. Consumers can meet multiple sellers and choose a seller with whom to trade. Consumers' preferences are given by private utility shocks, which they observe prior to trade but may or may not observe prior to seller choice. When consumers observe these shocks prior to seller choice, we call this *informed choice.* We calibrate the model to U.S. data and find that a greater degree of informed choice amplifies the negative welfare effects of inflation, making it significantly more costly.

JEL codes: D82, E31, E40, E50, E52
Keywords: Money; Inflation; Choice; Directed search; Private information

*We are grateful to Guillaume Rocheteau and Randy Wright for especially useful feedback. We also thank Jonathan Chiu, Michael Choi, Mohammad Davoodalhosseini, Lucas Herrenbrueck, Janet Jiang, Mario Silva, and Yu Zhu for useful comments, as well as seminar participants at the Bank of Canada, U.C. Irvine, Simon Fraser University, the National University of Singapore, and the Bank of Canada Summer Workshop on Money, Banking, and Payments.
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1 Introduction

In September 2022, core inflation in the U.S. reached 6.7%, its highest level since 1982.[1] Since the early 1980s, the nature of retail trade has changed radically as a result of various factors, including the rise of the internet. Less than 1% of the U.S. population used the internet in 1990 compared to almost 90% in 2019.[2] Not only are more purchases made online today but consumers have a greater ability to make *informed choices* about their purchases – both online and in store – due to the increased availability of online information about brands and product features. This raises the following question: Does a greater degree of informed choice by consumers make economies more or less vulnerable to the negative effects of inflation?

To answer this question, we build on the theoretical framework developed in a companion paper, Mangin (2023), which features both consumer choice and monetary exchange. We generalize this model by incorporating a parameter that represents the *degree of informed choice* available to consumers. This allows us to vary the extent to which consumers can make informed choices about which goods to purchase. We can therefore ask a precise question: How does the welfare cost of inflation vary with changes in the extent of consumers' informed choices?

We find that a greater degree of consumer choice significantly increases the welfare cost of inflation. As a result, economies in which buyers are more likely to be able to make informed choices – for example, as a result of rising internet availability – may be more sensitive to the negative effects of of inflation. This means that the same inflation rate may be more costly today than in earlier decades.

Search-theoretic models are widely used to model the microfoundations of monetary exchange, as discussed in the survey by Lagos, Rocheteau, and Wright (2017). However, these models do not generally feature what we call *consumer choice*, i.e. buyers' choice of seller. Typically, each buyer meets at most one seller during a period of time and chooses to either trade or wait.

We extend the model of Mangin (2023), which introduces consumer choice into the monetary framework of Rocheteau and Wright (2005). This framework is based on the Lagos and Wright (2005) alternating markets structure and it also features

[1] U.S. Bureau of Labor Statistics, Consumer Price Index for All Urban Consumers: All Items in U.S. City Average [CPIAUCSL], retrieved from FRED.

[2] World Bank, Internet Users for the United States [ITNETUSERP2USA], retrieved from FRED.

endogenous seller entry. As in Mangin (2023), we focus on *competitive search equilibrium.* Buyers and sellers choose to enter submarkets in which terms of trade, or contracts, are posted by market makers. Upon entry to each submarket, buyers and sellers commit to trading at the posted terms of trade. Within each submarket, search frictions exist that govern how buyers and sellers meet.

Directed or competitive search is a natural alternative to bargaining in our environment because buyers can meet multiple sellers within a single meeting. It is also a natural benchmark for welfare analysis because directed or competitive search is often used to decentralize the constrained efficient allocation in search-theoretic environments, as discussed in Wright, Kircher, Julien, and Guerrieri (2021). Moreover, since the cost of inflation is generally much lower when prices are determined by competitive search instead of bargaining, our estimates of the cost of inflation are conservative and can be interpreted as lower bounds.

As in Mangin (2023), we model search frictions within submarkets using a meeting technology that features *many-on-one meetings.* While the meeting technology in Mangin (2023) is general, we restrict attention to the Poisson meeting technology. During any given period of time, each seller meets exactly *one* buyer, but a buyer can meet either no sellers, one seller, or more than one seller.

After a meeting takes place, nature draws an i.i.d. preference or utility shock specific to each seller in the meeting. The buyer then chooses to purchase from the seller that maximizes their net utility. Prior to trade, sellers cannot observe buyers' utility shocks; they are private information for the buyer.

In order to answer our question regarding how changes in the degree of consumer choice affect the welfare cost of inflation, we introduce a new feature. Buyer's choice of seller is influenced by the information available to the buyer at the time this choice is made. While buyers always observe their utility shocks prior to trade, buyers may or may not observe these shocks prior to choosing a seller. We allow for two possibilities regarding the buyer's information. With probability $\pi \in (0, 1]$, buyers observe the seller-specific utility shocks *before* choosing a seller. In such cases, we say that buyers make an *informed choice* of seller. With probability $1 - \pi$, buyers observe the shock *after* choosing a seller but prior to trade. In such cases, buyers simply randomize across sellers. This flexibility allows us to examine the effect of a change in the extent of consumer choice, i.e. the *degree of informed choice* π.

As in Mangin (2023), one important consequence of consumer choice is that the distribution of utility shocks of *chosen* goods is endogenous and depends on the seller-buyer ratio. More sellers per buyer means that buyers can choose from a greater number of sellers (on average). With informed choice, this increases the average quality of the goods that are actually chosen by buyers in equilibrium, but with randomization there is no effect on the average quality of chosen goods. As a result, both the average quality of a chosen good and the average surplus depend directly on the seller-buyer ratio and also on the degree of informed choice π.

After buyers choose a seller with whom to trade, they choose the quantity of the good to purchase and make the corresponding payment. We focus on incentive-compatible direct revelation mechanisms that induce buyers to reveal their private information to their chosen seller. We establish the existence and uniqueness of equilibrium *for any degree of informed choice* $\pi \in (0, 1]$. There is only one active submarket in equilibrium, in which all sellers offer the same non-linear price schedule. For every realization of the buyer's utility shock, the price schedule specifies both the quantity traded and the corresponding payment in real dollars. Within any meeting, trades may or may not be liquidity constrained. Buyers may spend all of their money, some of their money, or none of their money.

After presenting our analytic results, we quantify the effect of consumer choice on the welfare cost of inflation. We calibrate the model to match data from Lucas and Nicolini (2015) on money demand in the U.S. from 1915-2008. For our baseline calibration, we target a retail markup of 30% as in Berentsen, Menzio, and Wright (2011), which implies a degree of choice $\pi = 0.54$. That is, 54% of all meetings are ones in which consumers make an informed choice of seller.

We estimate that the welfare cost of going from 0% to 10% inflation is equivalent to 0.93% of consumption at our baseline calibration. To determine the effect of consumer choice on the welfare cost of inflation, we vary the degree of choice π and recalibrate the model using the same calibration strategy for the other parameters. In particular, we compare results for the full choice calibration (i.e. $\pi = 1$) and the random choice calibration (i.e. the limit as $\pi \to 0$). We estimate that the cost of increasing inflation from 0% to 10% is more than twice as high with full choice: 1.45% of consumption compared to 0.61% with random choice. Moreover, we find that the cost of inflation is strictly increasing in the degree of choice π.

An alternative way to measure the welfare cost of inflation is to ask: What level of inflation leads to a welfare cost of 1% (compared to 0% inflation)? At our baseline calibration, this inflation rate is 11%. With full choice, this inflation rate is 7%, and with random choice, this inflation rate is 28%. This suggests that while consumers are better off in economies that feature a greater degree of informed choice, they are significantly more vulnerable to experiencing the negative welfare effects of inflation.

In our model, consumer choice makes inflation more costly because it *amplifies* the negative effects of inflation. With random choice, inflation is costly because buyers hold less money when inflation is higher, which leads to lower quantities traded and lower entry of sellers, which reduces the number of trades. When there is consumer choice, all of these effects continue to hold. However, there is an additional effect of inflation: lower seller entry directly reduces the average quality of chosen goods, which affects welfare by reducing the average match surplus directly (as well as indirectly through quantities). This is because the distribution of chosen goods is endogenous and depends on the seller-buyer ratio when there is choice. In turn, the effect of inflation on the distribution of chosen goods amplifies the negative effects of inflation on money holdings, quantities traded, and seller entry.

Outline. Section 2 discusses the related literature. Section 3 describes the model. Section 4 solves the planner's problem. Section 5 describes competitive search equilibrium and establishes existence and uniqueness of equilibrium. Section 6 presents our baseline calibration and some comparative statics. Section 7 provides our estimates of the cost of inflation. Section 8 describes the results of some robustness exercises. Section 9 concludes. The Appendix contains our random choice and full choice calibrations. All proofs are in the Online Appendix.

2 Related literature

There is a large literature on the welfare cost of inflation. Rocheteau and Nosal (2017) provides a summary of estimates of the welfare cost of 10% inflation, which vary from 0.2% to 7.2% of consumption. Cooley and Hansen (1989) estimates the cost of 10% inflation is less than 0.5% of consumption, while Lucas (2000) estimates that it is less than 1%. Lagos and Wright (2005) finds that the cost of 10% inflation is between 3% and 5% of consumption in a monetary model with search and bargaining. In competitive search equilibrium, the cost of inflation is typically much lower than under bargaining, e.g. Rocheteau and Wright (2009) estimates the cost of 10% inflation is between 0.67% and 1.1% of consumption.[3] Recently, Bethune, Choi, and Wright (2020) obtains a relatively low estimate for the cost of inflation – around 1% or less – by identifying a positive market-composition effect of inflation.

In a related paper featuring monetary exchange in a search model without private information, Dong (2010) considers the effect of product variety on the welfare cost of inflation when firms can invest to expand product variety. Greater product variety increases welfare by increasing the *probability* of trade in bilateral meetings. In contrast to our finding that the effect of consumer choice on the welfare cost of inflation is significant, Dong (2010) finds that the effect of endogenous product variety on the cost of inflation is negligible in competitive search equilibrium.

This paper is also related to the literature on directed and competitive search. For a survey, see Wright et al. (2021). In particular, we contribute to the literature on directed or competitive search and private information, including Faig and Jerez (2005), Menzio (2007), Guerrieri (2008), Guerrieri, Shimer, and Wright (2010), Moen and Rosen (2011), and Davoodalhosseini (2019). This paper is also related to a literature that features many-on-one or multilateral meetings in monetary environments include Julien, Kennes, and King (2008) and Galenianos and Kircher (2008). Finally, this paper is closely related to a literature that consider monetary environments featuring private information including Ennis (2008) and Faig and Jerez (2006), which builds on Faig and Jerez (2005), and Dong and Jiang (2014), and Choi and Rocheteau (2021). For a more detailed discussion of the relationship between our theoretical model and these papers, see Mangin (2023).

While our model extends the theoretical framework in the companion paper Mangin (2023), a theoretical contribution of this paper is to generalize this framework by incorporating a parameter that represents the *degree of informed choice* available to consumers. The generalized model in this paper is rich enough to allow us to vary the extent to which consumers can make informed choices prior to trading. This generalization is important because it enables us to ask how the welfare cost of inflation varies with changes in the extent of consumers' informed choices – the key question considered here. To maintain tractability, we restrict attention to a Poisson meeting technology for buyers and sellers, while Mangin (2023) considers

[3] Rocheteau and Wright (2009) use a slightly different formulation to calibrate the model in Rocheteau and Wright (2005). Instead of seller entry, agents can decide whether to be buyers or sellers.

the broader class of invariant meeting technologies described in Lester, Visschers, and Wolthoff (2015).

In the present paper, buyers may or may not observe their utility shocks prior to their choice of seller, allowing us to nest both informed choice and random choice. This distinction between informed choice and random choice by buyers is reminiscent of the distinction between informed and uninformed buyers in Lester (2011). However, the meaning of the term "informed" is different here. In our model, *all* buyers observe price schedules and engage in directed or competitive search when choosing submarkets, but *within meetings* buyers can either make an informed choice of seller (i.e. observe utility shocks prior to choosing a seller) or not.

3 Model

Each time period $t \in \{0, 1, 2, ...\}$ is divided into two subperiods, day and night, as in Lagos and Wright (2005). During the day, there is a frictionless, centralized market and at night there is a frictional, decentralized market.

There is a continuum of agents who are either ex-ante identical *buyers* or ex-ante identical *sellers*. During the day all agents both produce and consume, but at night only buyers wish to consume but cannot produce, and sellers do not wish to consume but can produce.

The measure of buyers is fixed and equal to one. All buyers participate in the night market at zero cost, but sellers must decide whether to enter and pay a cost $K > 0$. A subset of sellers of measure $n_t \in \mathbb{R}_+$ enter. Since the measure of buyers is one, n_t is also the seller-buyer ratio.

The aggregate money supply at date t is $M_t \in \mathbb{R}_+$, which grows at a constant rate $\gamma \in \mathbb{R}_+$, i.e. $M_{t+1} = \gamma M_t$. Money is either injected into the economy ($\gamma > 1$) or withdrawn ($\gamma < 1$) by lump sum transfers during the day. We assume these transfers are to buyers only, and we restrict attention to policies where $\gamma \geq \beta$, where β is the discount factor. At the Friedman rule, where $\gamma = \beta$, we consider equilibria obtained by taking the limit as $\gamma \to \beta$ from above.

The price of goods in the day market is normalized to one and the relative price of money is denoted by ϕ_t. The aggregate real money supply is $Z_t \equiv \phi_t M_t$, and the real value of a quantity m_t of money held by an agent is $z_t \equiv \phi_t m_t$.

We focus on steady-state equilibria where all of the aggregate real variables are constant. In steady state $\phi_{t+1}/\phi_t = 1/\gamma$, since $M_{t+1}/M_t = \gamma$.

Prices in the night market are determined in competitive search equilibrium, which we discuss in Section 5.

Many-on-one meetings. While all sellers meet exactly one buyer, a buyer can meet either no sellers, one seller, or more than one seller. We assume that the probability a buyer meets $k \in \{0, 1, 2, ...\}$ sellers is given by the Poisson distribution, i.e. $P_k(n) = \frac{n^k e^{-n}}{k!}$ for all k. The endogenous probability $\alpha(n)$ that a buyer has the opportunity to trade equals the probability that the buyer meets at least one seller, i.e. $\alpha(n) = 1 - e^{-n}$. Since all sellers meet exactly one buyer, the probability that a seller has the opportunity to trade equals $\alpha(n)/n$.

Buyer's information. After a meeting takes place, nature draws a seller-specific random utility shock a for each seller the buyer meets. The buyer then chooses a single seller with whom to trade in that subperiod.

There are two different possibilities with respect to buyers' information. With probability $\pi \in (0, 1]$, the buyer observes their utility shocks *before* choosing a seller. With probability $1 - \pi$, the buyer observes their shock *after* choosing a seller but before trade occurs. In the first case, we say that the buyer makes an *informed choice.* In the second case, buyers simply randomize across sellers. We refer to π as the *degree of choice*. We sometimes refer to the case where $\pi = 1$ as *full choice* and the case where $\pi \in (0, 1)$ as *partial choice*. We refer to the limiting case where $\pi \to 0$ as *random choice* since it is effectively equivalent to a model with bilateral meetings and random matching within submarkets.

Distribution of utility shocks. The random utility shocks a are drawn from a bounded distribution with cdf G that is known to all agents. The realizations of the utility shocks are private information for the buyer. We assume that G is not degenerate and Assumption 1 is maintained throughout the paper.

Assumption 1. *The distribution of utility shocks has a twice-differentiable cdf G, where $G' > 0$ and $G'' < 0$, and bounded support $A = [a_0, \bar{a}] \subseteq \mathbb{R}_+$.*

Taking $\pi \in (0, 1]$ as given, for any $n \in \mathbb{R}_+$ this distribution has cdf $\tilde{G}_\pi : A \to [0, 1]$, which depends on both the equilibrium seller-buyer ratio n and the equilibrium choices made by buyers. For brevity, we refer to G simply as the *distribution of available goods* and $\tilde{G}_\pi$ as the *distribution of chosen goods.*

Buyer and seller utility. On demand, sellers can produce any quantity $q \in \mathbb{R}_+$ of a divisible good and the cost of production is $c(q)$, where $c : \mathbb{R}_+ \to \mathbb{R}_+$. We assume that $c(0) = 0$, $c'(q) > 0$, and $c''(q) \geq 0$ for all $q > 0$. A buyer who consumes quantity q of a good with utility shock a receives utility $au(q)$, where $u : \mathbb{R}_+ \to \mathbb{R}_+$. We assume that $u(0) = 0$, $u'(0) = \infty$, $u'(q) > 0$, and $u''(q) < 0$ for all $q > 0$.

The instantaneous utility of a buyer who meets a seller at night at date t is

$$U_t^b = \nu(x_t) - y_t + \beta E_{\tilde{G}_{\pi,t}}(au(q_{a,t})), \tag{1}$$

and the instantaneous utility of a seller who is chosen by a buyer at night at date t is

$$U_t^s = \nu(x_t) - y_t - \beta E_{\tilde{G}_{\pi,t}}(c(q_{a,t})), \tag{2}$$

where x_t is the quantity consumed and y_t is the quantity produced during the day, $q_{a,t}$ is the quantity consumed at night, a is the *utility shock* of the good consumed, and $\tilde{G}_{\pi,t}$ is the distribution of *chosen goods* at time t. We assume $\nu'(x) > 0$ and $\nu''(x) < 0$ for all x, and that there exists x^* such that $\nu'(x^*) = 1$.

For now, we normalize $\nu(x^*) - x^* = 0$. Later, when we calibrate the model in Section 6, we will reverse this normalization.

4 Planner's problem

Before studying competitive search equilibrium, we briefly summarize the solution to the planner's problem. Suppose the planner is constrained by the same search frictions and meeting technology as the decentralized market. In addition, suppose that the planner faces the same information about utility shocks as buyers, i.e. the same probability $\pi \in (0, 1]$ of observing these shocks prior to choosing a seller with whom the buyer will trade.

Given the cost of seller entry $K > 0$, the planner chooses a seller-buyer ratio n^*, a function $q^* : A \to \mathbb{R}_+$, and a distribution of chosen goods $\tilde{G}_\pi : A \to [0, 1]$ to maximize the total surplus created minus the total cost of seller entry, subject to the above constraints regarding search frictions and information. The planner solves the following problem:

$$\max_{n\in\mathbb{R}_+,\{q_a\}_{a\in A}} \left\{\alpha(n)\int_{a_0}^{\bar{a}} [au(q_a)-c(q_a)]\, d\tilde{G}_\pi(a;n) - nK\right\} \tag{3}$$

where $\tilde{G}_\pi$ represents the planner's optimal choice of seller for each buyer.

Define $s_a \equiv au(q_a)-c(q_a)$, the trade surplus (or match surplus) for a good of quality a. Let q_a^* denote the efficient quantity of good a and define $s_a^* \equiv au(q_a^*) - c(q_a^*)$. Assume there is (weakly) positive trade surplus at the lowest utility a_0. Define the *expected trade surplus* by

$$\tilde{s}(n;\{q_a\}_{a\in A}) \equiv \int_{a_0}^{\bar{a}} [au(q_a)-c(q_a)] d\tilde{G}_\pi(a;n). \tag{4}$$

Throughout the paper, we sometimes suppress the dependence on $\{q_a\}_{a\in A}$ and let $\tilde{s}(n)$ denote $\tilde{s}(n;\{q_a\}_{a\in A})$ and $\tilde{s}'(n)$ denote $\partial\tilde{s}(n)/\partial n$.

Assumption 2 ensures the existence of a social optimum where $n^* > 0$.[4]

Assumption 2. *The cost of entry is not too high:* $E_G[au(q_a^*)-c(q_a^*)] > K$.

Before presenting the planner's solution, we derive the endogenous distribution of chosen goods. The *average quality* of a chosen good is defined by $\tilde{a}(n) \equiv E_{\tilde{G}_\pi}(a)$, i.e. $\tilde{a}(n) = \int_{a_0}^{\bar{a}} a d\tilde{G}_\pi(a;n)$. Lemma 1 states that the average quality of a *chosen* good $\tilde{a}(n)$ is greater than the average quality of an *available* good, $E_G(a)$. Moreover, Part 6 of Lemma 1 implies that $\tilde{a}(n)$ is strictly increasing in n, i.e. $\tilde{a}'(n) > 0$. Intuitively, average quality is increasing in the seller-buyer ratio because more sellers per buyer means greater choice of seller and a higher expected quality of the chosen good.

Lemma 1. *Suppose that the seller-buyer ratio $n > 0$. For any $\pi \in (0,1]$,*

1. *The distribution of chosen goods is given by*

$$\tilde{G}_\pi(a;n) = \pi\left(\frac{e^{-n(1-G(a))} - e^{-n}}{1-e^{-n}}\right) + (1-\pi)G(a). \tag{5}$$

2. *In the limit as $n \to 0$, we have $\tilde{G}_\pi(a;n) \to G(a)$ and $\tilde{a}(n) \to E_G(a)$.*

[4] Note that it follows from our previous assumptions that, for all $a \in A$, there exists a unique $q_a^* \in \mathbb{R}_+$ such that $au'(q_a^*) = c'(q_a^*)$.

3. *In the limit as $n \to \infty$, we have $\tilde{G}_\pi(a;n) \to (1-\pi)G(a)$ for all $a \in [a_0, \bar{a})$ and $\tilde{a}(n) \to \pi\bar{a} + (1-\pi)E_G(a)$.*
4. *The distribution of chosen goods $\tilde{G}_\pi(a;n)$ first-order stochastically dominates the distribution of available goods $G(a)$ and $\tilde{a}(n) > E_G(a)$.*
5. *If $n_1 > n_2$, the distribution $\tilde{G}_\pi(a;n_1)$ first-order stochastically dominates the distribution $\tilde{G}_\pi(a;n_2)$ and $\tilde{a}(n_1) > \tilde{a}(n_2)$.*
6. *For any $f : A \to \mathbb{R}_+$ where $f' > 0$, $\tilde{f}'(n) > 0$ where $\tilde{f}(n) \equiv \int_{a_0}^{\bar{a}} f(a) d\tilde{G}_\pi(a;n)$.*

Proposition 1 states that there exists a unique social optimum $(n^*, \{q_a^*\}_{a\in A})$ with $n^* > 0$ and provides the necessary conditions for an efficient allocation.

Proposition 1. *There exists a unique social optimum $(n^*, \{q_a^*\}_{a\in A})$ and it satisfies:*

1. *For any $a \in A$, the quantity $q_a^* > 0$ solves*

$$au'(q_a^*) = c'(q_a^*). \tag{6}$$

2. *The seller-buyer ratio $n^* > 0$ satisfies*

$$\alpha'(n^*)\tilde{s}(n^*; \{q_a^*\}_{a\in A}) + \alpha(n^*)\tilde{s}'(n^*; \{q_a^*\}_{a\in A}) = K. \tag{7}$$

3. *For any $\pi \in (0, 1]$, the distribution of chosen goods is given by (5).*

As in Mangin (2023), equation (7) can be rearranged to give the *generalized Hosios condition* derived in Mangin and Julien (2021). Defining the *matching elasticity* by $\eta_\alpha(n) \equiv \alpha'(n)n/\alpha(n)$ and the *surplus elasticity* by $\eta_s(n) \equiv \tilde{s}'(n)n/\tilde{s}(n)$, (7) says

$$\underbrace{\eta_\alpha(n)}_{\text{matching elasticity}} + \underbrace{\eta_s(n; \{q_a\}_{a\in A})}_{\text{surplus elasticity}} = \underbrace{\frac{nK}{\alpha(n)\tilde{s}(n; \{q_a\}_{a\in A})}}_{\text{sellers' surplus share}}. \tag{8}$$

Compared to Mangin (2023), the main difference is that the endogenous distribution of chosen goods now depends on the degree of informed choice π. Therefore, equation (7) also depends directly on the parameter π through $\tilde{s}(n; \{q_a\}_{a\in A})$. In the limit as $\pi \to 0$, we obtain the standard Hosios (1990) condition, which states that entry is constrained efficient only if sellers' surplus share equals the matching elasticity.

5 Competitive search equilibrium

Following Rocheteau and Wright (2005), we assume there are agents called "market makers" who can open submarkets by posting terms of trade. Specifically, market makers post contracts $\{(q_a, d_a)\}_{a \in A}$ which specify the quantity of the good q_a and the payment in real dollars d_a *contingent on the buyer's utility shock for their chosen seller.* Market makers take into account the relationship between the posted terms of trade and the expected seller-buyer ratio n.

Buyers and sellers choose which *submarket* to enter. Buyers and sellers who enter a submarket *commit* to trading at the terms specified within that submarket. Within each submarket, there are search frictions. Meetings take place, buyers choose sellers within meetings, and trade occurs as described in Section 3.

Within matches between buyers and their chosen seller, buyers' utility shocks are private information which is not observed by the seller. However, buyers may choose to reveal their private utility shocks to their chosen seller through their choice of contract (q_a, d_a). By the revelation principle, it is without loss of generality to focus on incentive-compatible direct mechanisms $\{(q_a, d_a)\}_{a \in A}$ that induce buyers to truthfully reveal their private information to their chosen sellers.

At the beginning of each day, market makers post submarkets $\{(q_a, d_a)\}_{a \in A}$ that will be open that night, which implies an expected n for each submarket. During the day, agents adjust their real money holdings in the centralized market, and then choose a submarket in which to trade at night, taking into account the expect seller-buyer ratio n in that submarket. At night, agents trade in the decentralized market in their chosen submarket.

Let W^b and W^s denote the value functions for buyers and sellers, respectively, in the day market and let V^b and V^s denote the value functions for buyers and sellers, respectively, in the night market.

Centralized market. In the CM, a buyer with real money holdings z solves:

$$W^b(z) = \max_{\hat{z}, x, y \in \mathbb{R}_+} \{\nu(x) - y + \beta V^b(\hat{z})\}, \tag{9}$$

subject to $\hat{z} + x = z + T + y$, where T is her real transfer and $\hat{z}$ is the real balance carried into that period's decentralized market. Substituting into (9) yields

$$W^b(z) = z + T + \max_{\hat{z}, x \in \mathbb{R}_+} \{\nu(x) - x - \hat{z} + \beta V^b(\hat{z})\}. \tag{10}$$

Thus, the buyer's $\hat{z}$ is independent of z, and $W^b(z) = z + W^b(0)$, which is linear.

Similarly, a seller with real balance z_s in the centralized market solves:

$$W^s(z_s) = \max_{\hat{z}, x, y \in \mathbb{R}_+} \left\{ \nu(x) - y + \beta \max \left[V^s(\hat{z}), W^s \left(\frac{\hat{z}}{\gamma} \right) \right] \right\}, \tag{11}$$

subject to $\hat{z} + x = z_s + y$. Substituting into (11), we obtain

$$W^s(z_s) = z_s + \max_{\hat{z}, x \in \mathbb{R}_+} \left\{ \nu(x) - x - \hat{z} + \beta \max \left[V^s(\hat{z}), W^s \left(\frac{\hat{z}}{\gamma} \right) \right] \right\}. \tag{12}$$

Thus, the seller's $\hat{z}$ is independent of z_s, and $W^s(z_s) = z_s + W^s(0)$.

Decentralized market. The value functions V^b and V^s depend on the distribution of chosen goods, which is endogenous and depends on both the degree of informed choice π and the seller-buyer ratio n. For any $\pi \in (0, 1]$, the equilibrium distribution of chosen goods $\tilde{G}_\pi$ is given by buyers' optimal choices of sellers. In any meeting, the buyer chooses the seller that maximizes $v_a \equiv au(q_a) - d_a/\gamma$, the buyer's ex post trade surplus. We derive this distribution later.

Let Ω denote the set of open submarkets, where each submarket $\omega \in \Omega$ is characterized by $(\{(q_a, d_a)\}_{a \in A}, n)_\omega$. For a seller in the decentralized night market,

$$V^s(z_s) = \max_{\omega \in \Omega} \left\{ \begin{array}{c} \frac{\alpha(n)}{n} \int_{a_0}^{\bar{a}} \left[-c(q_a) + W^s \left(\frac{z_s + d_a}{\gamma} \right) \right] d\tilde{G}_\pi(a; n) \\ + \left[1 - \frac{\alpha(n)}{n} \right] W^s \left(\frac{z_s}{\gamma} \right) - K \end{array} \right\} \tag{13}$$

where each submarket $\omega \in \Omega$ is characterized by $(\{(q_a, d_a)\}_{a \in A}, n)$. It is easy to show that the seller's choice of real balances is $\hat{z} = 0$.

For a buyer in the decentralized night market,

$$V^b(z) = \max_{\omega \in \Omega} \left\{ \begin{array}{c} \alpha(n) \int_{a_0}^{\bar{a}} \mathbf{1}_a \left[au(q_a) + W^b \left(\frac{z - d_a}{\gamma} \right) \right] d\tilde{G}_\pi(a; n) \\ + \left[1 - \alpha(n) \int_{a_0}^{\bar{a}} \mathbf{1}_a \, d\tilde{G}_\pi(a; n) \right] W^b \left(\frac{z}{\gamma} \right) \end{array} \right\} \tag{14}$$

where $\mathbf{1}_a$ is an indicator function that is equal to one if $z \geq d_a$ and zero otherwise. Using $W^b(z) = z + W^b(0)$ we obtain

$$V^b(z) = \max_{\omega \in \Omega} \left\{ \alpha(n) \int_{a_0}^{\bar{a}} \mathbf{1}_a \left[au(q_a) - \frac{d_a}{\gamma} \right] d\tilde{G}_\pi(a;n) + \frac{z}{\gamma} + W^b(0) \right\}. \tag{15}$$

Thus, the buyer's choice of z from (10) is given by

$$\max_{z \in \mathbb{R}_+} \left\{ -z + \beta \max_{\omega \in \Omega} \left\{ \alpha(n) \int_{a_0}^{\bar{a}} \left[au(q_a) - \frac{d_a}{\gamma} \right] d\tilde{G}_\pi(a;n) + \frac{z}{\gamma} \right\} \right\} \tag{16}$$

subject to the liquidity constraint, $d_a \leq z$ for all $a \in A$.

Defining $i \equiv \frac{\gamma - \beta}{\beta}$, the nominal interest rate, the above problem is equivalent to

$$\max_{z \in \mathbb{R}_+,\, \omega \in \Omega} \left\{ \alpha(n) \int_{a_0}^{\bar{a}} \left[au(q_a) - \frac{d_a}{\gamma} \right] d\tilde{G}_\pi(a;n) - i\frac{z}{\gamma} \right\}, \tag{17}$$

subject to $d_a \leq z$ for all $a \in A$ plus the constraint that a submarket with posted contracts $\{(q_a, d_a)\}_{a \in A}$ will attract measure n of sellers per buyer, where n satisfies

$$\frac{\alpha(n)}{n} \int_{a_0}^{\bar{a}} \left[-c(q_a) + \frac{d_a}{\gamma} \right] d\tilde{G}_\pi(a;n) \leq K \tag{18}$$

and $n \geq 0$ with complementary slackness.

5.1 Existence, uniqueness, and characterization

As a result of buyers' private information, we need to impose two additional constraints on problem (17). The individual rationality (IR) constraint states that buyers must receive a (weakly) positive ex post trade surplus, or trade will not occur. The IR constraint for buyers is given by

$$au(q_a) - \frac{d_a}{\gamma} \geq 0 \tag{19}$$

for all $a \in A$. The incentive compatibility (IC) constraint states that a buyer with utility shock a cannot do better by choosing another contract $(q_{a'}, d_{a'})$ instead of

(q_a, d_a). The IC constraint is given by

$$au(q_a) - \frac{d_a}{\gamma} \geq au(q_{a'}) - \frac{d_{a'}}{\gamma} \tag{20}$$

for all $a, a' \in A$.

Our definition of competitive search equilibrium is identical to that in Mangin (2023) except for the *degree of informed choice* π. We restrict attention to steady-state monetary equilibria where $z > 0$ and $n > 0$. We later prove that there is a unique solution to the market makers' problem and there is therefore only one active submarket in equilibrium. In anticipation of this result, we denote equilibrium by $(\{(q_a, d_a)\}_{a\in A}, z, n)$ and define it as follows.

Definition 1. *For any degree of choice* $\pi \in (0, 1]$*, a* competitive search equilibrium *is a list* $(\{(q_a, d_a)\}_{a\in A}, z, n)$ *and a distribution of chosen goods* $\{\tilde{G}_\pi(a; n)\}_{a\in A}$ *where* $(q_a, d_a) \in \mathbb{R}^2_+$ *for all* $a \in A$*,* $\tilde{G}_\pi(a; n) \in [0, 1]$ *for all* $a \in A$*, and* $z, n \in \mathbb{R}_+\backslash\{0\}$*, such that* $(\{(q_a, d_a)\}_{a\in A}, z, n)$ *maximizes (17) subject to constraint (18), the liquidity constraint* $d_a \leq z$ *for all* $a \in A$*, plus the IR constraint (19) and the IC constraint (20), and* $\{\tilde{G}_\pi(a; n)\}_{a\in A}$ *represents buyers' optimal choices of sellers.*

Before presenting Proposition 2, it is helpful to define $\rho(a; n) \equiv 1 - \tilde{G}_\pi(a; n)$, the probability that a chosen good has quality greater than a. We also define $\varepsilon_\rho(a; n) \equiv -a\rho'(a; n)/\rho(a; n)$, the elasticity of $\rho(a; n)$ with respect to a, where $\rho'(a; n) \equiv \frac{\partial \rho(a;n)}{\partial a}$. This elasticity can be calculated as follows:

$$\varepsilon_\rho(a; n) \ = \frac{a\tilde{g}_\pi(a; n)}{1 - \tilde{G}_\pi(a; n)}. \tag{21}$$

For simplicity, we assume $a_0 = 0$ for the remainder of the paper. We also assume that the virtual valuation function is strictly increasing, a condition known as *regularity* in the mechanism design literature. Note that this condition is weaker than both the increasing hazard rate condition and log-concavity.

Assumption 3. *The distribution is regular, i.e.* $\psi'_G(a) > 0$ *where*

$$\psi_G(a) \equiv a - \frac{1 - G(a)}{g(a)}. \tag{22}$$

We can now present our main result, which establishes the existence and uniqueness of equilibrium and provides a characterization.

For existence, we need to ensure that the entry cost K is not "too high".

Assumption 4. *The cost of entry is not too high:* $E_G[au(q_a^0) - c(q_a^0)] > K$.

In the Online Appendix, we make this condition precise by showing how to calculate $q_a^0 \equiv \lim_{n\to 0} q_a(n)$ in terms of the distribution of utility shocks G.

Proposition 2. *For any* $\pi \in (0, 1]$ *and* $i > 0$, *if the cost of entry* K *is not too high then there exists a unique competitive search equilibrium and it satisfies:*

1. *No-trade range. For any* $a \in [a_0, a_b]$, $q_a = 0$ *and* $d_a = 0$.

2. *Unconstrained range. For any* $a \in (a_b, a_c]$, *the quantity* $q_a > 0$ *solves:*

$$(a - \phi(a; n))u'(q_a) = c'(q_a) \tag{23}$$

where

$$\phi(a; n) = \left(1 - \frac{1}{\delta}\right)\left(\frac{1 - \tilde{G}_\pi(a; n)}{\tilde{g}_\pi(a; n)}\right) - \left(\frac{1}{\delta}\right)\frac{i}{\alpha(n)\tilde{g}_\pi(a; n)} \tag{24}$$

and

$$\delta = \frac{1}{1 - \varepsilon_\rho(a_b; n)}\left(1 + \frac{i}{\alpha(n)\rho(a_b; n)}\right). \tag{25}$$

Also, $d_a/\gamma = au(q_a) - \int_{a_0}^{a} u(q_x)dx$.

3. *Liquidity constrained range. For any* $a \in [a_c, \bar{a}]$, $q_a = q_{a_c}$ *and* $d_a = d_{a_c}$.

4. *The value of* a_c *satisfies*

$$\frac{i\bar{a}}{\alpha(n)} = \int_{a_c}^{\bar{a}} (a - a_c)\tilde{g}(a; n)dx + (\delta - 1)(\bar{a} - a_c)(1 - \tilde{G}(a_c; n)). \tag{26}$$

5. *Real money holdings* $z > 0$ *is given by* $z = d_{a_c}$.

6. *The seller-buyer ratio* $n > 0$ *is strictly decreasing in* K *and satisfies*

$$\alpha'(n)\tilde{s}(n; \{q_a\}_{a\in A}) + \alpha(n)\tilde{s}'(n; \{q_a\}_{a\in A}) = K. \tag{27}$$

7. The zero profit condition is satisfied:

$$\frac{\alpha(n)}{n}\int_{a_0}^{\bar{a}}\left[-c(q_a)+\frac{d_a}{\gamma}\right]d\tilde{G}_\pi(a;n)=K. \tag{28}$$

8. For any $\pi \in (0,1]$, the distribution of chosen goods is given by

$$\tilde{G}_\pi(a;n)=\pi\left(\frac{e^{-n(1-G(a))}-e^{-n}}{1-e^{-n}}\right)+(1-\pi)G(a). \tag{29}$$

Due to buyers' private information, there exists a non-empty range of utility shocks a such that trade does not occur in equilibrium, i.e. $q_a = 0$. When the good chosen by a buyer within a meeting falls within this range, we call such meetings *no-trade meetings*. Due to the liquidity constraint, there exists a non-empty range of utility shocks such that buyers' purchases are constrained by their money holdings, i.e. $d_a = z$. When the good chosen by a buyer within a meeting falls within this range, we call such meetings *liquidity constrained*.

The equilibrium distribution of chosen goods $\tilde{G}_\pi$ is the same as the planner's. With probability π the buyer can observe the utility shocks a prior to choosing a seller. In this case, buyers always choose the highest quality seller they meet. The distribution of chosen goods therefore equals the distribution across buyers of the highest quality a among the sellers a buyer meets, conditional on meeting at least one seller. With probability $1-\pi$, the buyer observes the shock only after choosing a seller. In this case, buyers randomize across the sellers they meet. The distribution of chosen goods is therefore equal to the distribution of available goods. In general, for any degree of informed choice $\pi \in (0,1]$, the cdf of the equilibrium distribution $\tilde{G}_\pi$ is a weighted average of these two possibilities. In the limiting case of random choice where $\pi \to 0$, we have $\tilde{G}_\pi \to G$.

In terms of efficiency, as discussed in Mangin (2023), there are two margins: an *extensive margin* (seller entry) and an *intensive margin* (quantity traded). With consumer choice, the extensive margin has two components since seller entry directly affects both the number of trades *and* the expected trade surplus. For any $\pi \in (0,1]$, there may be inefficiencies on both the intensive and extensive margins. In particular, outside the Friedman rule, there are various possibilities for ranges of underconsumption and overconsumption relative to the efficient quantity, and there may be either under-entry or over-entry of sellers, as in Mangin (2023).

The Friedman rule does not deliver efficiency along *either* the extensive or the intensive margin. For any degree of informed choice $\pi \in (0,1]$, there is underconsumption of all goods. Second, for any $\pi \in (0,1]$, there may be either under-entry, over-entry, or efficient entry of sellers at the Friedman rule.

These inefficiencies at the Friedman rule are due to the presence of private information. As discussed in Mangin (2023), we do obtain constrained efficiency in the case of full information, i.e. when a buyer's chosen seller can observe the buyer's utility shocks prior to trade. We discuss this case further in Section 8.1.

6 Calibration

We calibrate the model to match the data from Lucas and Nicolini (2015) on money demand in the U.S. from 1915-2008.[5] The period length is set to one year. We set $\beta = 1/(1+r)$ to match a real interest rate of $r = 0.03$ as in Bethune et al. (2020). We use the 3-month U.S. T-bill rate as a measure of the nominal interest rate i. The average nominal interest rate i for the period 1915-2008 is $i = 0.0383$. Money demand $L(i)$ is defined as $M1/GDP$.

In the model, money demand is $L(i) = z/Y$ where z is real money holdings and Y is real GDP given by $Y = x^* + \alpha(n)\tilde{d}(n)$, where x^* is the quantity consumed in the CM, $\tilde{d}(n) \equiv \int_{a_0}^{\bar{a}} \frac{d_a}{\gamma} d\tilde{G}_\pi(a;n)$, the average payment for a chosen good, and $\alpha(n) = 1 - e^{-n}$, the probability a buyer has the opportunity to trade.

We assume that $c(q) = q$ and $u(q) = \frac{(q+\epsilon)^{1-\sigma} - \epsilon^{1-\sigma}}{1-\sigma}$ where $\sigma \in (0,1)$ and $\epsilon \approx 0$. The CM utility function is $\nu(x) = A \log x$. Since $\nu'(x^*) = 1$, we have $x^* = A$.

We assume the distribution of utility shocks has cdf $G(a) = \left(\frac{a}{\bar{a}}\right)^\psi$ on $[a_0, \bar{a}]$ where $\bar{a} \in \mathbb{R}_+ \backslash \{0\}$ and $\psi \geq 1$. This is a Beta distribution that satisfies Assumption 3 provided that $\psi \geq 1$.[6] To ensure that a decrease in ψ is a mean-preserving spread of G, we normalize $\bar{a} = \frac{\psi+1}{2\psi}$ so that $E_G(a) = 0.5$ regardless of ψ.[7]

For our baseline calibration, we focus on the case where $\psi = 1$. In this case, the distribution G is uniform on $[0,1]$. We focus on this case for two reasons. First, it is a standard benchmark. Second, we find that this distribution generates price

[5]Lucas and Nicolini (2015) adjust the measure of $M1$ to generate a stable money demand curve.

[6]While the assumption that $G''(a) \leq 0$ in Assumption 1 is not satisfied, we verify that $q'(a) \geq 0$ directly. Recall that $G''(a) \leq 0$ is sufficient but not necessary for existence of equilibrium.

[7]In general, the expected value of this distribution is $E_G(x) = \psi\bar{a}/(\psi+1)$.

dispersion that is reasonable, as we discuss below. In Section 8.3, we provide a robustness exercise where we vary the dispersion of G via the parameter ψ.

Parameter		*Target*	
DM utility curvature, $1-\sigma$	0.719	elasticity of money demand, η_L	-0.16
CM utility parameter, A	1.99	average money demand, $L(i)$	0.272
cost of entry, K	0.0184	buyers' surplus share, $\theta(n)$	0.50
degree of choice, π	0.54	decentralized market markup, μ_{DM}	1.30

Table 1: Baseline calibration

Baseline calibration. For our baseline calibration, we calibrate four parameters (A, σ, K, π) to match four targets. The target for the steady state level of money demand in the model, $L(i)$ where $i = 0.0383$, is equal to 0.272, the average money demand in the data for 1915-2008. The target for the elasticity of money demand $L(i)$ with respect to i, denoted by η_L, is equal to -0.16, the elasticity in the data for 1915-2008. Our third target is *buyers' surplus share*, defined by $\theta(n) \equiv \tilde{v}(n)/\tilde{s}(n)$, where $\tilde{v}(n) \equiv \int_{a_0}^{\bar{a}} v_a d\tilde{G}_\pi(a;n)$ and $v_a \equiv au(q_a) - \frac{d_a}{\gamma}$. We treat $\theta(n)$ as a proxy for buyers' bargaining power and target $\theta(n) = 0.5$.[8] Our fourth target is the *markup* in the decentralized market. Defining average quantity by $\tilde{q}(n) \equiv \int_{a_0}^{\bar{a}} q_a d\tilde{G}_\pi(a;n)$, the average unit price is $\tilde{p}(n) \equiv \tilde{d}(n)/\tilde{q}(n)$. The DM markup is defined by $\mu_{DM} \equiv \tilde{p}(n)/c'(q)$, which is equal to the average price $\tilde{p}(n)$ since we assume $c(q) = q$ for our calibration. We follow Berentsen et al. (2011) in targeting a DM markup of $\mu_{DM} = 1.3$ to reflect a retail markup of 30%.

Discussion of calibration strategy. In the monetary search literature featuring bargaining, buyers' bargaining power is a parameter and it is generally calibrated to match either the DM markup or the aggregate markup. Since prices are determined in competitive search equilibrium in our model, we cannot do this because buyers' surplus share is endogenous. However, it is important to ensure that we fix buyers' surplus share when we compare calibrations for different values of π in Section 8 because the cost of inflation depends strongly on buyers' surplus share, as discussed in Craig and Rocheteau (2008). Our strategy is to ensure that we "fix" buyers' surplus share at steady state through our choice of K and match the DM markup through our choice of π. Given that we can match the DM markup as a

[8]Note that $\theta(n) = \theta$ is the value of buyer's surplus share that would deliver the same buyer/seller shares as the Kalai (proportional) bargaining solution with parameter θ.

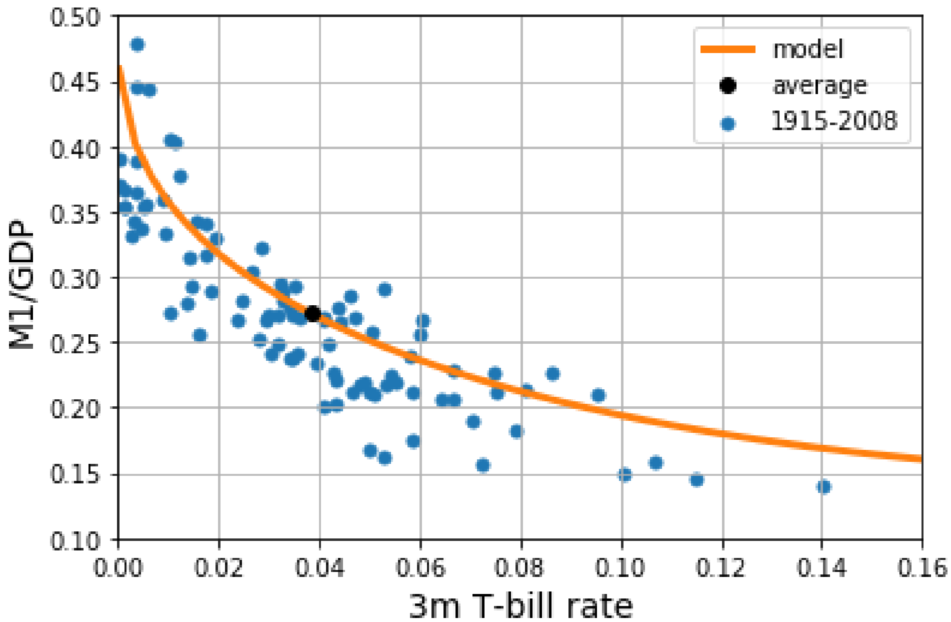

Figure 1: Data vs model predictions for money demand (by nominal interest rate i)

separate target, the choice of target for buyers' surplus share is somewhat arbitrary, hence we simply set $\theta(n) = 0.5$. In Section 8.2, we provide a robustness exercise where we vary this target and show that our main result is preserved.

Table 2 provides a summary of the equilibrium outcomes for our baseline calibration. The equilibrium features underconsumption of goods of *all* qualities (i.e. there is no overconsumption). The equilibrium is also *partial trade*: around 23% of meetings do not result in any trade. Around 33% of meetings and 26% of trades are liquidity constrained. Buyers spend around 41% of their money holdings on average.

We do not target the output share of the decentralized market, but it is around 9%.[9] We also do not target price dispersion, but it is close to the empirical estimates in Kaplan and Menzio (2015). Defining unit prices by $p_a \equiv \frac{d_a/\gamma}{q_a}$ for all traded goods (i.e. DM markup since $c'(q) = 1$), *price dispersion* is defined as the standard deviation of normalized prices across all trades.[10] Price dispersion is 25% at our baseline calibration, which fits well within the range of empirical estimates, 19% to 36%, found in Table 2 of Kaplan and Menzio (2015) and is equal to their estimate of

[9]In the literature, values of the DM output share vary from less than 10% in Lagos and Wright (2005) to 25% in Bethune et al. (2020) and 42% in Berentsen et al. (2011).

[10]Standard deviations are expressed as a percentage of the mean throughout the paper.

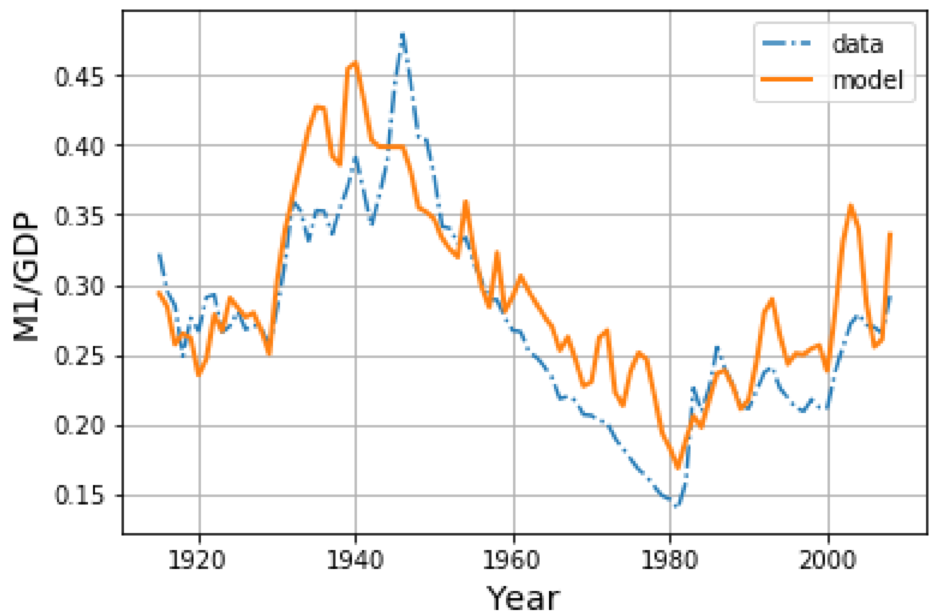

Figure 2: Data vs model predictions for money demand (by year)

25% for the broader definition of goods which aggregates brands (but not sizes).[11]

Comparative statics. We provide some comparative statics results for the cost of entry K, the inflation rate $\tau \equiv \gamma - 1$, and the degree of choice π. Table 2 summarizes the effects of a 1% increase in the parameters K, $\gamma \equiv 1 + \tau$, and π from our baseline calibration. In Table 2, we can see that greater informed choice among buyers increases seller entry, increases the average quality of a chosen good, and increases the average quantity of goods purchased. Greater choice also increases money holdings and the average payment, as well as increasing the average size of the trade surplus. Buyers' surplus share does not change by much when we increase the degree of choice by 1%, but it decreases slightly at the baseline calibration. The average price or DM markup also decreases slightly at baseline. Total real output and welfare (as defined below in Section 7) are both increasing in the degree of informed choice at baseline. Appendix C contains some figures to illustrate the comparative statics over a wider range of parameter values.

[11]We believe the "brand aggregation" measure in Kaplan and Menzio (2015) is the most relevant since goods are not strictly identical in our environment where consumers experience idiosyncratic utility or preference shocks that differ across goods.

	Baseline	$1+\tau$ ($\uparrow$ inflation)	K ($\uparrow$ cost)	π ($\uparrow$ choice)
seller-buyer ratio, n	3.08	-2.3%	-1.1%	0.9%
meeting prob, $\alpha(n)$	0.95	-0.4%	-0.2%	0.1%
average quality, $\tilde{a}(n)$	0.62	-0.4%	-0.2%	0.3%
average quantity, $\tilde{q}(n)$	0.20	-7.3%	-0.7%	0.9%
average payment, $\tilde{d}(n)$	0.26	-6.1%	-0.5%	0.9%
money holdings, z/γ	0.60	-8.5%	0.1%	0.3%
average surplus, $\tilde{s}(n)$	0.12	-2.6%	-0.5%	0.7%
buyer share, $\theta(n)$	0.50	-0.6%	-0.6%	-0.0%
price or markup, $\tilde{p}(n)$	1.30	1.3%	0.2%	-0.0%
price dispersion	0.25	1.1%	0.6%	-0.1%
total real output, Y	2.23	-0.7%	-0.1%	0.1%
total welfare, W	0.43	-0.5%	-0.2%	0.1%

Table 2: Equilibrium outcomes and comparative statics at baseline calibration

7 Welfare cost of inflation

In this section, we present our estimates of the welfare cost of inflation and show how it varies with the degree of informed choice by consumers. We start by defining total welfare in economy E by[12]

$$W(E) = \alpha(n) \int_{a_0}^{\bar{a}} [au(q_a) - c(q_a)] d\tilde{G}_\pi(a;n) - nK + \nu(x^*) - x^* + 1. \tag{30}$$

Since consumers' utility depends on both quality and quantity, in order to calculate the consumption sacrifice in terms of quantity alone we first convert to a welfare-equivalent "representative" economy in which the quantity of goods traded is constant and quality is normalized to one. That is, we find quantity q such that

$$W(E) = \alpha(n)[u(q) - c(q)] - nK + \nu(x^*) - x^* + 1. \tag{31}$$

If total consumption is multiplied by a factor of $\Delta \in [0,1]$, then welfare is given by

$$W(E, \Delta) = \alpha(n)[u(\Delta q) - c(q)] - nK + \nu(\Delta x^*) - x^* + 1. \tag{32}$$

[12]Note that adding one is a normalization that ensures $W(E)$ is positive for all calibrations we consider. It does not affect our estimates of the welfare cost of inflation.

We measure the welfare cost of moving from economy E to E' by the share of total consumption that consumers are willing to give up in order to go from economy E' to E. That is, the cost is $1 - \Delta$ where $\Delta \in [0, 1]$ satisfies $W(E, \Delta) = W(E')$.

We compute the welfare cost of 10% inflation relative to both 0% inflation and the Friedman rule. In particular, we find $\Delta_0 \in [0, 1]$ such that $W(\gamma = 1, \Delta_0)$ is equal to $W(\gamma = 1.1, \Delta = 1)$. The value $1 - \Delta_0$ is the percentage of total consumption that consumers are willing to give up in order to go from 10% inflation to 0% inflation. We also find $\Delta_F \in [0, 1]$ such that $W(\gamma = \beta, \Delta_F)$ is equal to $W(\gamma = 1.1, \Delta = 1)$. The value $1 - \Delta_F$ is the percentage of total consumption that consumers are willing to give up in order to go from 10% inflation to the Friedman rule.

7.1 How does consumer choice affect the cost of inflation?

As we would expect, consumer choice increases the level of welfare. Starting at our random choice calibration ($\pi \to 0$), we estimate that increasing the degree of choice to our baseline level ($\pi = 0.54$) delivers a welfare gain worth 1.58% of total consumption. Similarly, starting at our baseline degree of choice ($\pi = 0.54$), an increase in the degree of choice to $\pi = 1$ delivers a welfare gain worth 2.75% of total consumption. The positive effect of greater choice on welfare is intuitive. A greater degree of informed choice by consumers increases both the average quality and the average quantity traded, as well as increasing seller entry. However, the effect of choice on the welfare cost of inflation is not clear.

Figure 3 illustrates how the welfare cost of inflation varies with the degree of choice π. For any given value of π, we recalibrate the parameters (A, σ, K) to match the first three targets of our baseline calibration. Figure 3 shows that the cost of inflation is strictly increasing in the degree of consumer choice π.

Table 3 provides our estimates of the welfare cost of inflation. Recall that $1 - \Delta_0$ (or $1 - \Delta_F$) denotes the welfare cost of moving from 0% (or the Friedman rule) to 10% inflation. We focus on comparing our baseline calibration ($\pi = 0.54$), full choice calibration ($\pi = 1$), and random choice calibration ($\pi \to 0$). Details of the full choice and random choice calibrations can be found in Appendix A.

At our baseline calibration ($\pi = 0.54$), the cost of increasing inflation from 0% to 10% is 0.93% of consumption, while the cost of moving from the Friedman rule to 10% inflation is 1.11% of consumption. When we recalibrate the model after

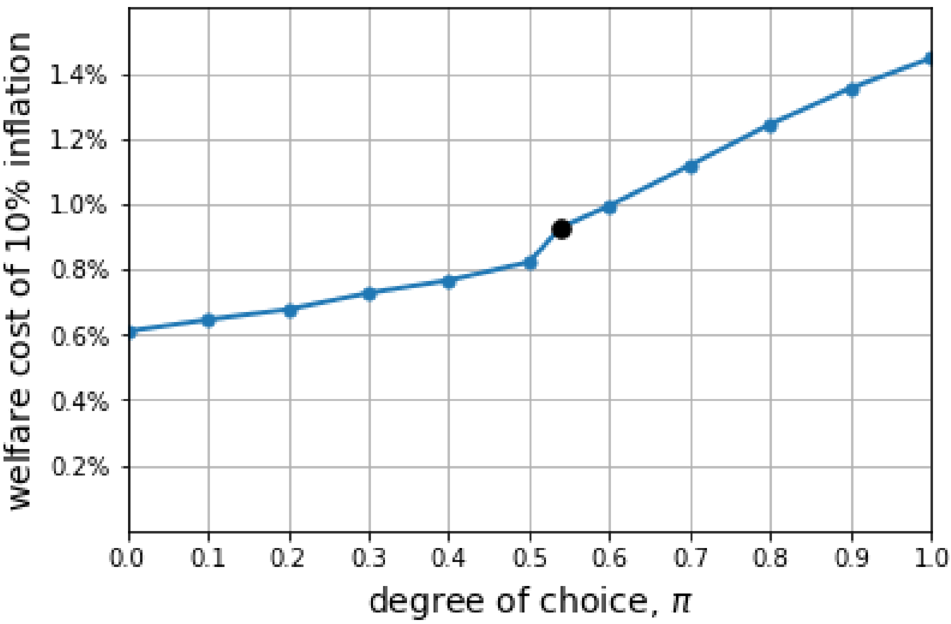

Figure 3: Welfare cost of 0% to 10% inflation for different values of π (recalibrated)

	$1-\Delta_0$	$1-\Delta_F$
Random ($\pi = 0$)	0.61%	0.79%
Baseline ($\pi = 0.54$)	0.93%	1.11%
Full choice ($\pi = 1$)	1.45%	1.64%

Table 3: Welfare cost of inflation (baseline, full choice, and random choice)

imposing random choice ($\pi \to 0$), the welfare cost of increasing inflation from 0% to 10% is 0.61% of consumption and the cost of moving from the Friedman rule to 10% inflation is 0.79% of consumption. On the other hand, when we recalibrate the model with full choice ($\pi = 1$), the cost of increasing inflation from 0% to 10% is more than twice as high at 1.45% of consumption, while the cost of moving from the Friedman rule to 10% inflation is 1.64% of consumption.[13]

Figure 4 depicts the welfare cost of increasing inflation from 0% to τ for various inflation rates τ at our baseline, random choice, and full choice calibrations. We can see that a welfare cost of 1% of consumption requires an inflation rate of around

[13] In Section 8.4, we show that our main result – greater choice increases the cost of inflation – still holds when we vary π and adjust k to match the target surplus share, while keeping the utility parameters (A, σ) at their baseline levels. This confirms that the difference in the welfare cost is driven by variation in the degree of choice π, not by differences across calibrations in either the utility parameters (A, σ) or the buyer surplus share $\theta(n)$.

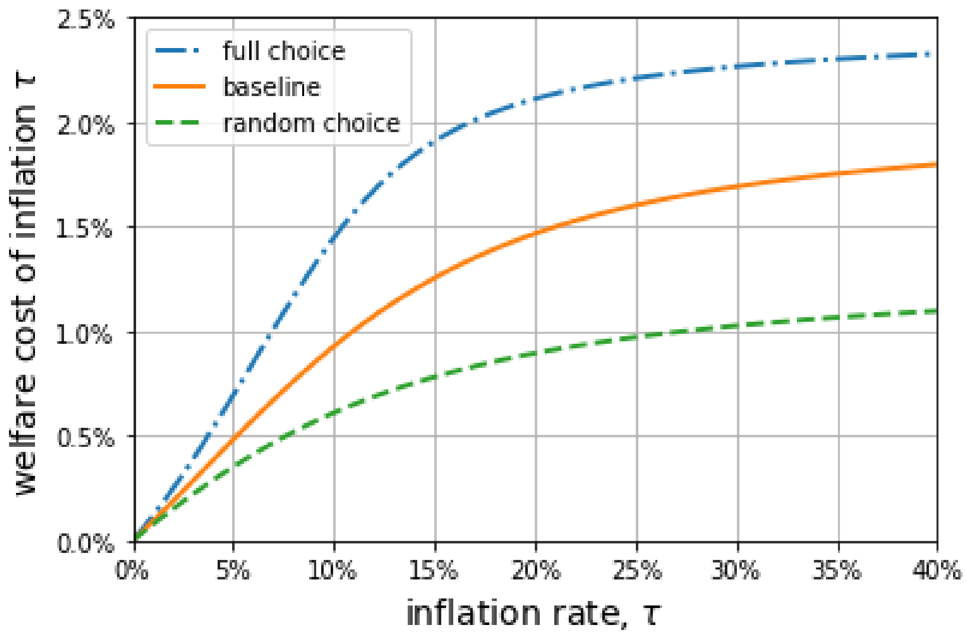

Figure 4: Welfare cost of 0% to τ inflation for different inflation rates τ

11% in our baseline calibration. With random choice, a very high inflation rate of around 28% is required for the same welfare cost. With full choice, a relatively low inflation rate of around 7% delivers the same welfare cost. This suggests that economies featuring a greater degree of informed choice can experience the same level of negative welfare effects from lower levels of inflation.

7.2 Why is the cost of inflation higher with consumer choice?

To understand better the negative effects of inflation on welfare in our model, Table 4 shows how the equilibrium outcomes change when the economy shifts from either the Friedman rule or 0% inflation to 10% inflation at the baseline calibration. We also include the efficient outcomes (given baseline $\pi = 0.54$) for comparison.[14]

When the economy shifts from 0% to 10% inflation, the seller-buyer ratio falls by 21.4%. As a result, the meeting probability for buyers falls and average quality drops by 3.4%. Money holdings fall dramatically by 48.8%, while average quantity traded decreases by 49.2%, average payment falls by 42.3%, and average surplus drops by 24.3%. As inflation jumps from 0% to 10%, buyers' surplus share falls by

[14]Notice that we have *over-entry* of sellers at the Friedman rule relative to the efficient allocation.

	Efficient	Friedman rule	0% inflation	10% inflation
seller-buyer ratio, n	3.26	3.28	3.13	2.46
meeting prob, $\alpha(n)$	0.96	0.96	0.96	0.91
average quality, $\tilde{a}(n)$	0.63	0.63	0.62	0.60
average quantity, $\tilde{q}(n)$	0.35	0.27	0.21	0.11
average payment, $\tilde{d}(n)$	-	0.33	0.27	0.16
money holdings, z/γ	-	1.13	0.65	0.33
average surplus, $\tilde{s}(n)$	0.14	0.13	0.12	0.09
buyer share, $\theta(n)$	-	0.51	0.50	0.46
price or markup, $\tilde{p}(n)$	-	1.23	1.28	1.46
price dispersion	-	0.24	0.25	0.27
total real output, Y	-	2.31	2.25	2.13
total welfare, W	0.45	0.44	0.44	0.42

Table 4: Equilibrium outcomes at different inflation rates (baseline calibration)

8.5%. The average price or DM markup rises by 13.7% and price dispersion rises by 9.6%. Total real output or GDP decreases by 5.2% and welfare falls by 4.5%.

The effects of inflation at our baseline calibration lie somewhere in between the effects at the two extremes of full choice ($\pi = 1$) and random choice ($\pi = 0$). To see how the sensitivity of various equilibrium outcomes to changes in inflation varies with the degree of choice π, Table 5 compares the comparative statics effect of a 1% increase in the parameter $1 + \tau$ (for inflation rate τ) for our three calibrations.

	Random ($\pi = 0$)	Baseline ($\pi = 0.54$)	Full choice ($\pi = 1$)
seller-buyer ratio, n	-1.4%	-2.3%	-3.0%
meeting prob, $\alpha(n)$	-0.7%	-0.4%	-0.0%
average quality, $\tilde{a}(n)$	0.0%	-0.4%	-0.5%
average quantity, $\tilde{q}(n)$	-6.1%	-7.3%	-9.2%
average payment, $\tilde{d}(n)$	-4.4%	-6.1%	-8.3%
money holdings, z/γ	-7.3%	-8.5%	-9.7%
average surplus, $\tilde{s}(n)$	-1.7%	-2.6%	-3.7%
buyer share, $\theta(n)$	-1.1%	-0.6%	-0.8%
price or markup, $\tilde{p}(n)$	1.8%	1.3%	1.0%
price dispersion	1.5%	1.1%	1.3%
total real output, Y	-0.3%	-0.7%	-1.6%
total welfare, W	-0.3%	-0.5%	-0.9%

Table 5: Effect of a 1% increase in $1 + \tau$ (inflation τ) for baseline, full choice, random

As Table 5 shows, greater choice *amplifies* the sensitivity of the economy to changes in inflation. First of all, it increases the sensitivity of seller entry to inflation. In response to a 1% increase in $1 + \tau$, the seller-buyer ratio falls by 3.0% with full choice compared to just 1.4% with random choice. Consumer choice also results in a higher sensitivity of average quality to changes in inflation, since average quality is unchanged with random choice. At the same time, average quantity, average payments and money holdings are also more sensitive to changes in inflation when there is greater choice. Finally, the sensitivity of average surplus to changes in inflation is amplified by greater choice. In response to a 1% increase in $1 + \tau$, average surplus falls by 3.7% with full choice compared to just 1.7% with random choice. The effects at baseline π fall somewhere in between these two extremes.

8 Robustness

In this section, we establish the robustness of our main result that the welfare cost of inflation is increasing in the degree of choice π. First, we consider how our results regarding the effect of choice on the welfare cost of inflation change when we shut down the private information. Second, we consider how our results change when we vary the target surplus share, which is $\theta(n) = 0.5$ for our baseline calibration. Third, we examine how our results change when we relax our assumption that the distribution of utility shocks G is uniform and vary the degree of *dispersion* of this distribution. Finally, we present the results of two different experiments where we shut down either endogenous seller entry or endogenous surplus shares.

8.1 Effect of private information

In order to examine the effect of private information on the welfare cost of inflation (in the presence of consumer choice), we consider an alternative model in which the chosen seller can directly observe the buyer's utility shock for that seller prior to trade. For brevity, we refer to this as *full information*. The necessary conditions for equilibrium under full information are found in Appendix B. Both with and without choice, the Friedman rule delivers constrained efficiency.

Table 6 presents the welfare cost of inflation for the full information version of our model with both random choice and full choice, and at the baseline calibration. When we shut down private information, the baseline π also varies and equals the value π_μ that matches the DM markup.

	Full info	Private info
Random ($\pi = 0$)	1.55%	0.61%
Baseline ($\pi = \pi_\mu$)	1.69%	0.93%
Full choice ($\pi = 1$)	2.61%	1.45%

Table 6: Welfare cost of 0% to 10% inflation for full information vs private information

It is clear that private information decreases the welfare cost of inflation. At the same time, private information *amplifies* the effect of consumer choice. That is, the difference in the welfare cost of inflation for different degrees of informed choice is higher when there is private information. For example, moving from random choice to the baseline degree of choice π_μ increases the cost of inflation by only 9% with full information compared to 52% with private information.

Importantly, our key result that consumer choice significantly increases the welfare cost of inflation still holds even when we shut down the private information between buyers and their chosen sellers.

8.2 Effect of surplus share target

Table 7 reports our estimates of the welfare cost of inflation when we vary the target value of buyers' surplus share and recalibrate the model using the same strategy (for baseline, full choice, and random choice). When we vary the target surplus share, baseline π also varies and equals the value π_μ that matches the DM markup. While our exact estimates of the cost of inflation depend on the target value of buyers' surplus share, it is clear from Table 7 that the cost of inflation is increasing in the degree of choice for every level of the target surplus share.

Since we focus on competitive search, our estimates of the cost of inflation can be viewed as lower bounds when compared to environments featuring bargaining. In such environments, the cost of inflation is sensitive to changes in the bargaining parameter. In Lagos and Wright (2005), the cost of inflation decreases as buyers' bargaining parameter θ increases because the severity of the hold-up problem decreases as $\theta \to 1$. In environments such as ours that feature competitive search, there is no hold-up problem. Buyers' surplus share is endogenous and depends crucially on the equilibrium seller-buyer ratio. As Table 7 shows, the cost of inflation actually *increases* in our model as we increase the target for buyers' surplus share.

	$\theta(n) = 0.4$	$\theta(n) = 0.5$	$\theta(n) = 0.6$
Random ($\pi = 0$)	0.25%	0.61%	0.81%
Baseline ($\pi = \pi_\mu$)	0.55%	0.93%	0.94%
Full choice ($\pi = 1$)	0.90%	1.45%	1.65%

Table 7: Welfare cost of 0% to 10% inflation for different values of target buyer share

8.3 Effect of dispersion of utility shocks

The welfare cost of inflation is sensitive to variation in the *dispersion* of the distribution of utility shocks G. To get a sense of how the parameter ψ of this distribution affects dispersion, Table 8 reports the standard deviation of the distribution G and the distribution of chosen goods $\tilde{G}$ (at our baseline degree of informed choice π). Table 8 also includes the surplus elasticity, which we discuss below.

	$\psi = 1$	$\psi = 2$	$\psi = 3$	$\psi = 4$	$\psi = 5$
Standard deviation of G	0.577	0.354	0.258	0.204	0.169
Standard deviation of $\tilde{G}$	0.457	0.298	0.217	0.170	0.140
Surplus elasticity, $\eta_s(n)$	0.35	0.23	0.20	0.18	0.16

Table 8: Effect of parameter ψ. *Note:* Standard deviations are relative to mean.

Table 9 reports our estimates of the cost of inflation when we vary the parameter ψ and recalibrate the model to match the same targets as our baseline calibration.

	$\psi = 1$	$\psi = 2$	$\psi = 3$	$\psi = 4$	$\psi = 5$
Random ($\pi = 0$)	0.61%	0.86%	1.05%	1.24%	1.43%
Baseline ($\pi = \pi_\mu$)	0.93%	1.17%	1.42%	1.66%	1.91%
Full choice ($\pi = 1$)	1.45%	1.66%	1.84%	2.02%	2.20%

Table 9: Welfare cost of 0% to 10% inflation for different values of parameter ψ.

While the welfare cost of inflation is increasing in ψ, our key result that consumer choice increases the cost of inflation is preserved when we vary the distribution of utility shocks. For every value of ψ, the cost of inflation remains higher with choice.

As ψ increases and the dispersion of utility shocks falls, the effect of consumer choice on the welfare cost of inflation reduces. In the limit as ψ becomes large, the distribution of utility shocks converges to a degenerate distribution and the gap in the welfare cost of inflation disappears. Intuitively, the presence of informed choice has no effect when buyers' utility does not differ across goods.

Surplus elasticity. The differences between the welfare cost of inflation with different degrees of informed choice depend largely on the value of the surplus elasticity, $\eta_s(n)$. In our calibration, we target a fixed value of buyers' surplus share, $\theta(n) = \theta$, and the generalized Hosios condition (8), which is equivalent to equilibrium condition (27), becomes $\eta_\alpha(n) + \eta_s(n) = 1 - \theta$ at steady state i. With random choice, $\eta_s(n) = 0$ and this condition reduces to the standard Hosios condition, $\eta_\alpha(n) = 1 - \theta$. As Table 9 shows, the effect of consumer choice on the welfare cost of inflation increases with the surplus elasticity $\eta_s(n)$. The impact of choice on the welfare cost of inflation is therefore driven by the extent to which the generalized Hosios condition differs from the standard Hosios condition.

Intuitively, the welfare cost of inflation is sensitive to the value of the elasticity $\eta_s(n)$ because it governs the sensitivity of the expected trade surplus $\tilde{s}(n)$ to changes in seller entry, n. With choice, the elasticity $\eta_s(n) > 0$ is driven by endogenous changes in the distribution of chosen goods $\tilde{G}(a; n)$ in response to seller entry. As the elasticity $\eta_s(n)$ decreases, the impact of choice diminishes and the gap in the welfare cost of inflation falls. In the limit as $\eta_s(n) \to 0$, this gap disappears.

8.4 Results of experiments

Our main result regarding the effect of choice on the welfare cost of inflation does not depend on either of two features of our model: (1) endogenous seller entry; and (2) endogenous surplus shares. To demonstrate this, we conduct two experiments. In the first experiment, we shut down endogenous seller entry by fixing the seller-buyer ratio to $n = \bar{n}$. In the second experiment, we shut down endogenous surplus shares by fixing buyers' surplus share to $\theta(n) = \bar{\theta}$.[15] For both experiments, the equilibrium conditions are the same as Proposition 2 except that entry cost K is replaced by endogenous J (where J is equal to equilibrium expected seller utility before entry cost). To calculate welfare, we use definition (30) and set $K = 0$.

Table 10 compares the cost of inflation for our main model and the experiments. We use the same calibration strategy as our main model except we treat $\bar{\theta}$ or $\bar{n}$ as a calibrated parameter (instead of K). For our main results (a) in Table 10, we

[15] In one sense, Experiment 2 is similar to the fixed surplus shares in a model featuring bargaining. However, it is different because we use competitive search and the equilibrium surplus shares are always the efficient ones (conditional on the quantities traded being efficient). This is why we still have relatively low costs of inflation in Experiment 2 compared to bargaining models.

recalibrate (A, σ) to match the money demand targets when we vary π (as for Table 3). We also include an additional check (b) where we keep (A, σ) at the baseline parameters when we vary π. For both (a) and (b), we match the surplus share target $\theta(n) = 0.5$ by adjusting K (or $\bar{\theta}$ or $\bar{n}$) to ensure comparability of welfare costs.

	Main model	Exper 1 $(n = \bar{n})$	Exper 2 $(\theta(n) = \bar{\theta})$
(a) Recalibrated (A, σ)			
Random $(\pi = 0)$	0.61%	0.58%	0.58%
Baseline $(\pi = 0.54)$	0.93%	0.87%	1.16%
Full choice $(\pi = 1)$	1.45%	1.22%	1.87%
(b) Baseline (A, σ)			
Random $(\pi = 0)$	0.52%	0.56%	0.56%
Baseline $(\pi = 0.54)$	0.93%	0.87%	1.16%
Full choice $(\pi = 1)$	1.14%	0.96%	1.53%

Table 10: Welfare cost of 0% to 10% inflation for main model and experiments

With random choice, the cost of inflation is the same for both experiments because fixing $\theta(n) = \bar{\theta}$ and fixing $n = \bar{n}$ are equivalent. This is because the standard Hosios condition applies under random choice, i.e. $\eta_\alpha(n) = 1 - \theta(n)$. However, when $\pi > 0$ and there is some degree of informed choice, the generalized Hosios condition applies, i.e. $\eta_\alpha(n) + \eta_s(n; \{q_a\}_{a \in A}) = 1 - \theta(n)$, and fixing the seller-buyer ratio is not equivalent to fixing the surplus shares. As a result, the welfare cost of inflation differs across these two experiments when there is consumer choice.

For both experiments, Table 10 (a) shows that our main result – greater choice increases the welfare cost of inflation – is confirmed when we vary π and recalibrate (A, σ) using the same strategy as our baseline calibration. For Experiment 1 (exogenous n), the cost of inflation is around twice as high with full choice compared to random choice. For Experiment 2 ($\theta(n)$ exogenous), the cost of inflation is more than three times as high with full choice compared to random choice.

Table 10 (b) shows that, for both our main model and both of our experiments, our result that greater choice increases the cost of inflation also holds when we vary π and adjust K to match the target surplus share, but keep the utility parameters (A, σ) equal to the baseline parameters. This confirms that, in all three cases, the difference in the welfare cost of inflation is not due to changes in the utility parameters (A, σ), or differences in the buyer surplus share $\theta(n)$, across calibrations for different π, but is instead due to variation in the degree of choice π.

9 Conclusion

This paper asks the following question: How does consumer choice affect the welfare cost of inflation? To answer this question, we develop a search-theoretic model of monetary exchange in which the *degree of informed choice* by consumers can vary. When we calibrate the model to U.S. data on money demand, we find that a greater degree of informed choice makes inflation significantly more costly. This suggests that while consumers benefit from the ability to make more informed choices about their purchases, this feature of an economy may also make consumers more vulnerable to the negative effects of inflation.

In future work, we believe it would be interesting to use our model to explore the implications of changes in the structure of retail trade – for example, the rise of online transactions and various online platforms – for monetary policy.

Appendix A: Full choice and random choice

For the full choice calibration, we set $\pi = 1$ and then calibrate the remaining three parameters (A, σ, K) to match the first three targets of our baseline calibration. Table 11 reports the calibrated parameters and targets.

Parameter		*Target*	
DM utility curvature, $1 - \sigma$	0.815	elasticity of money demand, η_L	-0.16
CM utility parameter, A	1.75	average money demand, $L(i)$	0.272
cost of entry, K	0.0081	buyers' surplus share, $\theta(n)$	0.50

Table 11: Full choice calibration ($\pi = 1$)

Table 12 summarizes the equilibrium outcomes and the comparative statics effects of a 1% increase in parameters K, $\gamma \equiv 1 + \tau$, and π for full choice.[16]

	Baseline	$1+\tau$ ($\uparrow$ inflation)	K ($\uparrow$ cost)	π ($\uparrow$ choice)
seller-buyer ratio, n	7.09	-3.0%	-0.9%	1.0%
meeting prob, $\alpha(n)$	1.00	-0.0%	-0.0%	0.0%
average quality, $\tilde{a}(n)$	0.86	-0.5%	-0.1%	0.6%
average quantity, $\tilde{q}(n)$	0.36	-9.2%	-0.8%	1.8%
average payment, $\tilde{d}(n)$	0.41	-8.3%	-0.7%	1.7%
money holdings, z/γ	0.58	-9.7%	-0.1%	0.5%
average surplus, $\tilde{s}(n)$	0.11	-3.7%	-0.5%	1.4%
buyer share, $\theta(n)$	0.50	-0.8%	-0.7%	0.4%
price or markup, $\tilde{p}(n)$	1.16	1.0%	0.1%	-0.1%
price dispersion	0.12	1.3%	1.0%	-1.0%
total real output, Y	2.16	-1.6%	-0.1%	0.3%
total welfare, W	0.29	-0.9%	-0.2%	0.4%

Table 12: Comparative statics for full choice calibration ($\pi = 1$)

For the random choice calibration, we let $\pi \to 0$. We then calibrate the remaining three parameters (A, σ, K) to match the first three targets of our baseline calibration. Table 13 reports the calibrated parameters and targets.

Table 14 summarizes the equilibrium outcomes and the comparative statics effects of a 1% increase in the parameters K, $\gamma \equiv 1 + \tau$, and π for random choice.

[16] Since $\pi = 1$ with full choice, we instead calculate the effect of a 1% *decrease* in π and then reverse the sign in Table 12.

Parameter		*Target*	
DM utility curvature, $1-\sigma$	0.641	elasticity of money demand, η_L	-0.16
CM utility parameter, A	2.06	average money demand, $L(i)$	0.272
cost of entry, K	0.0363	buyers' surplus share, $\theta(n)$	0.50

Table 13: Random choice calibration ($\pi \to 0$)

	Baseline	$1+\tau$ ($\uparrow$ inflation)	K ($\uparrow$ cost)	π ($\uparrow$ choice)
seller-buyer ratio, n	1.26	-1.4%	-0.9%	1.0%
meeting prob, $\alpha(n)$	0.72	-0.7%	-0.5%	0.5%
average quality, $\tilde{a}(n)$	0.50	0.0%	0.0%	0.2%
average quantity, $\tilde{q}(n)$	0.14	-6.1%	-0.4%	0.6%
average payment, $\tilde{d}(n)$	0.20	-4.4%	-0.1%	0.5%
money holdings, z/γ	0.60	-7.3%	0.4%	0.4%
average surplus, $\tilde{s}(n)$	0.13	-1.7%	-0.2%	0.5%
buyer share, $\theta(n)$	0.50	-1.1%	-0.7%	-0.1%
price or markup, $\tilde{p}(n)$	1.46	1.8%	0.3%	0.0%
price dispersion	0.37	1.5%	0.5%	0.0%
total real output, Y	2.21	-0.3%	0.0%	0.1%
total welfare, W	0.48	-0.3%	-0.1%	0.1%

Table 14: Comparative statics for random choice calibration ($\pi \to 0$)

With random choice, the direction of the effects is generally the same as with full choice, but the magnitude is often significantly lower. The only differences in direction are *(i)* average quality, which does not vary in the absence of choice; and *(ii)* money holdings, which are locally decreasing in entry cost with full choice, but increasing with random choice. At our baseline calibration with partial choice ($\pi = 0.54$), money holdings are locally increasing in entry cost, but non-monotonic (and decreasing over most of the parameter range) as Figure 6 shows.

Appendix B: Full information

With full information, there are three trading regions. In the first trading region, the IR constraint binds ($v_a = 0$) and sellers extract the full surplus, and a positive quantity $q_a > 0$ is traded. In the second trading region, both the IR constraint binds ($v_a = 0$) and the liquidity constraint (LC) binds ($d_a = z$). In the third trading region, the LC constraint binds ($d_a = z$), but buyers get positive ex post surplus ($v_a > 0$). It is straightforward to show that the Friedman rule delivers constrained efficiency of both entry and quantities traded with full information.[17]

Proposition 3. *Any full information competitive search equilibrium satisfies:*

1. *IR constraint binds. For $a \in [a_0, a_r]$, $d_a/\gamma = au(q_a)$ and $q_a > 0$ solves*

$$au'(q_a) = c'(q_a).$$

2. *IR and LC constraints bind. For $a \in [a_r, a_n]$, $d_a = z$ and $q_a > 0$ solves*

$$au(q_a) = d_a/\gamma.$$

3. *LC binds. For any $a \in (a_n, \bar{a}]$, $d_a = z$ and $q_a > 0$ solves:*

$$au'(q_a) = \left(1 + \frac{i}{\alpha(n)[1 - \tilde{G}(a_n; n)]}\right) c'(q_a). \tag{33}$$

4. *The seller-buyer ratio $n > 0$ satisfies*

$$\alpha'(n)\tilde{s}(n; \{q_a\}_{a \in [a_0, \bar{a}]}) + \alpha(n)\tilde{s}'(n; \{q_a\}_{a \in [a_0, \bar{a}]}) = k. \tag{34}$$

5. *The zero profit condition is satisfied:*

$$\frac{\alpha(n)}{n} \int_{a_0}^{\bar{a}} \left[-c(q_a) + \frac{d_a}{\gamma}\right] d\tilde{G}(a; n) = k. \tag{35}$$

6. *The distribution of chosen goods is given by (5).*

[17] These results are different from those described in Mangin (2023) because we impose the IR constraint for the full information case as it is quantitatively important for our results.

Appendix C: Comparative statics

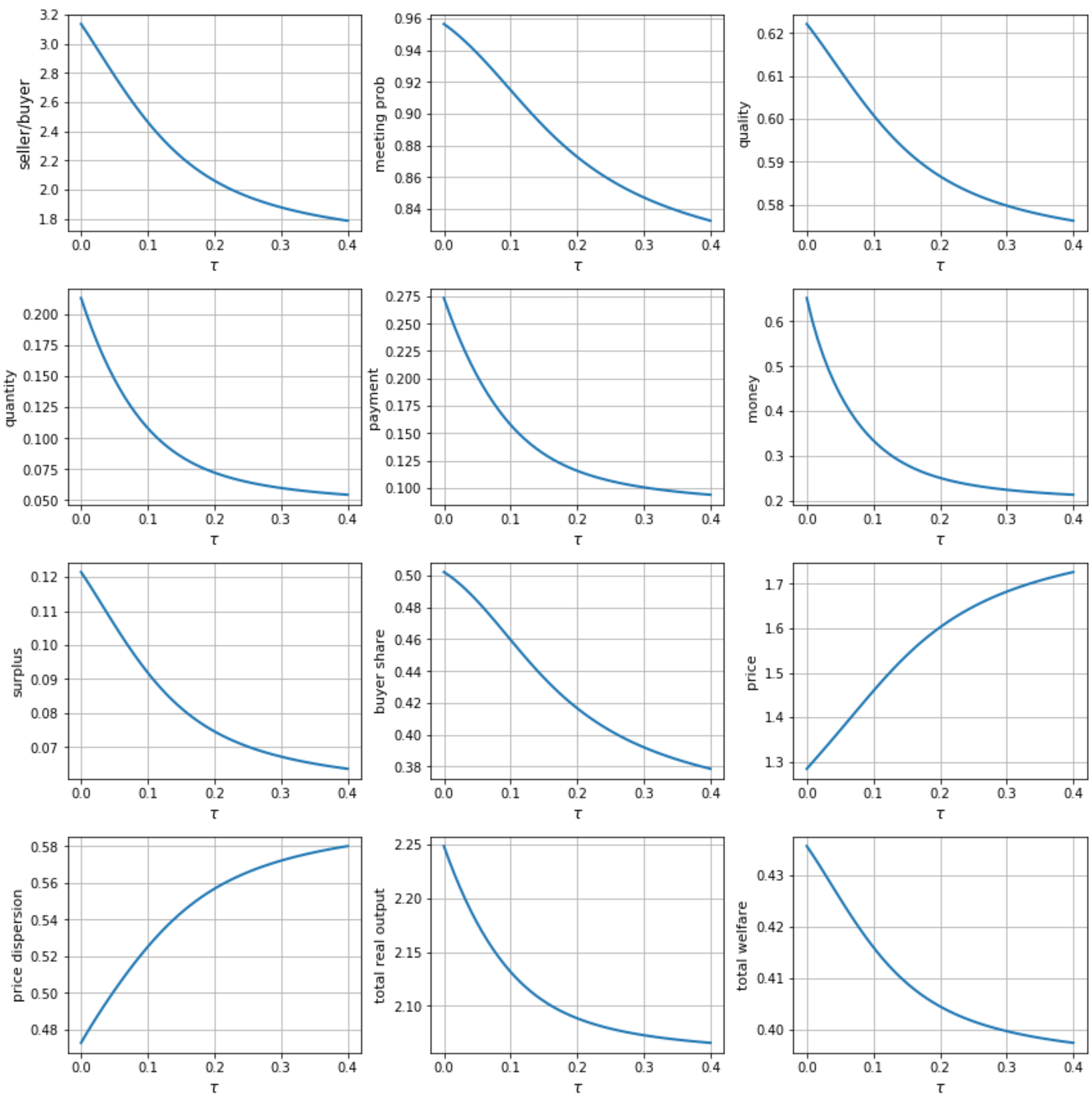

Figure 5: Comparative statics with respect to inflation rate τ

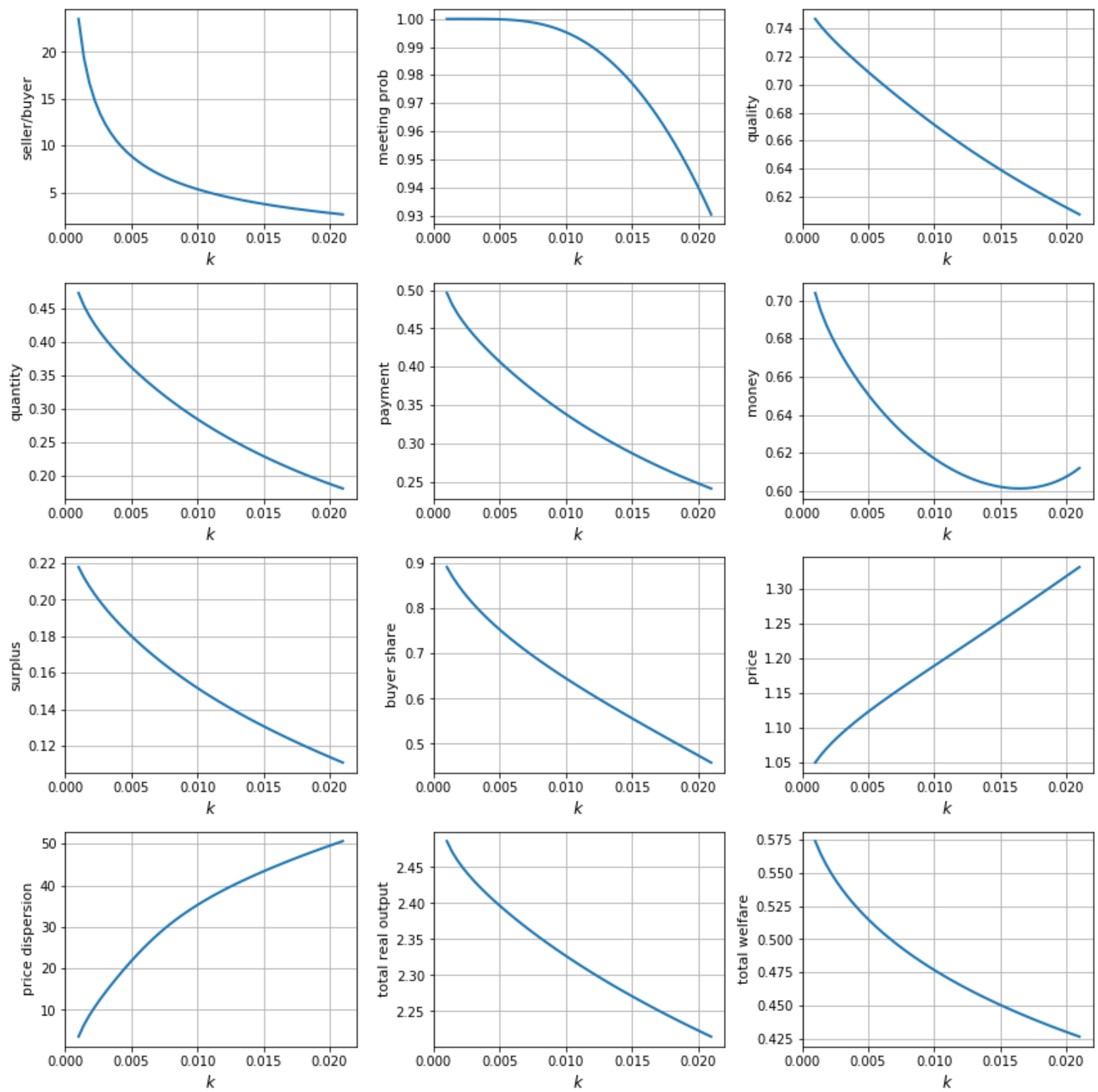

Figure 6: Comparative statics with respect to entry cost K

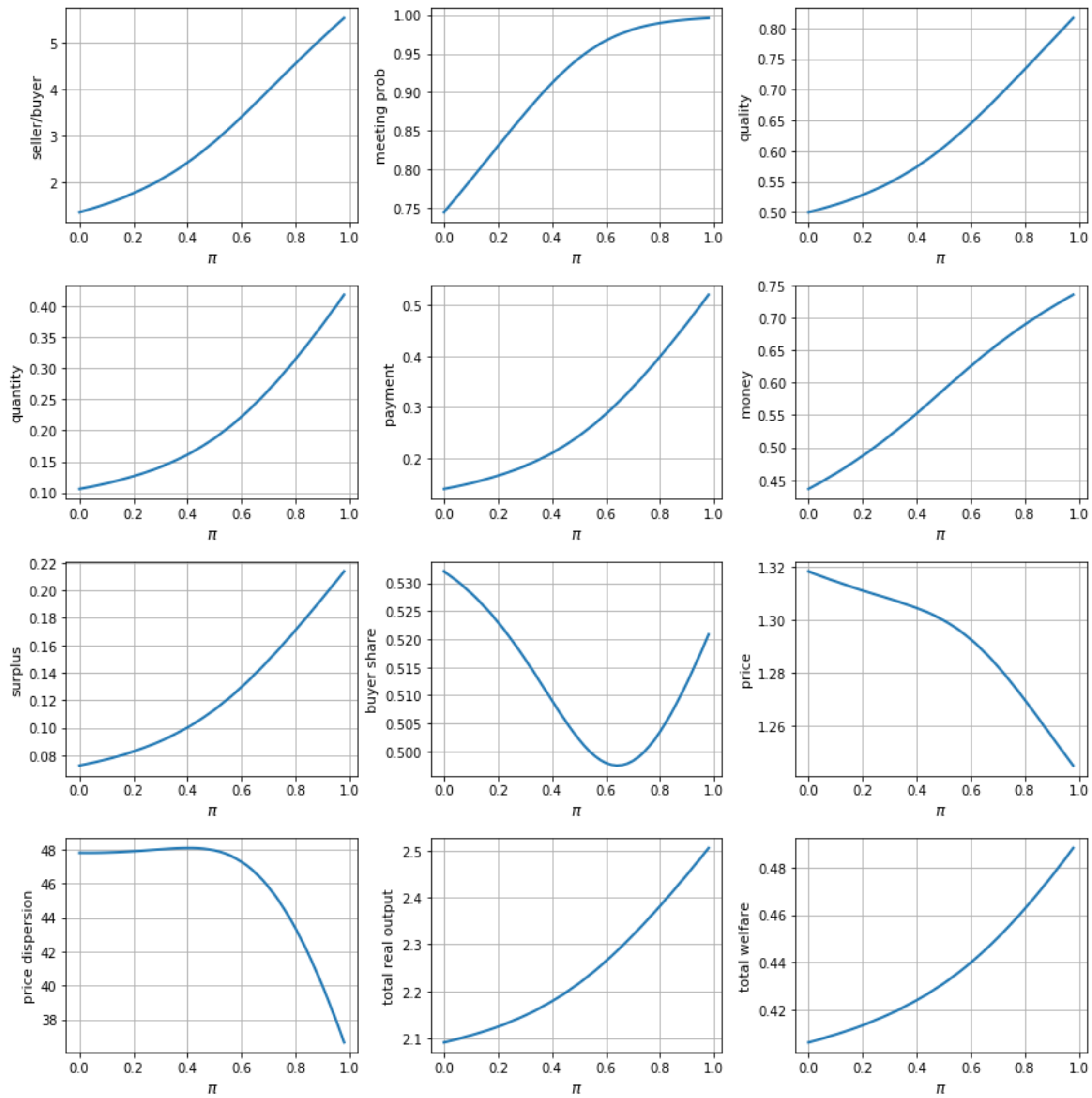

Figure 7: Comparative statics with respect to degree of choice π

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Online Appendix: Proofs

Proof of Lemma 1

Part 1. Since we assume the planner faces the same information as the buyer, with probability $\pi \in (0,1]$ the planner can observe the utility shocks a prior to choosing a seller. We verify in the proof of Proposition 1 that in this case the planner always chooses the seller with the highest utility shock among those the buyer meets. With probability $1-\pi$, the planner cannot observe the utility shocks prior to choosing a seller and they simply choose a seller at random.

Using the fact that the distribution of the maximum of $k \geq 1$ draws is $(G(a))^k$, and weighting by the probability $P_k(n)$ that exactly k sellers meet a buyer, conditional on $k \geq 1$, we obtain

$$\tilde{G}_\pi(a;n) = \frac{\pi \sum_{k=1}^{\infty} P_k(n)(G(a))^k}{\alpha(n)} + (1-\pi)G(a). \tag{36}$$

Given that we assume a Poisson distribution, substituting $P_k(n) = \frac{n^k e^{-n}}{k!}$ and $\alpha(n) = 1 - e^{-n}$ into the above yields

$$\tilde{G}_\pi(a;n) = \frac{\pi \left(e^{-n} \sum_{k=0}^{\infty} \frac{(nG(a))^k}{k!} - e^{-n} \right)}{1-e^{-n}} + (1-\pi)G(a) \tag{37}$$

which, using the fact that $\sum_{k=0}^{\infty} \frac{(nG(a))^k}{n!} = e^{-n(G(a))}$, simplifies to (5).

Part 2. Taking the limit as $n \to 0$, we have

$$\lim_{n\to 0} \tilde{G}_\pi(a;n) = \pi \lim_{n\to 0} \left(\frac{e^{-n(1-G(a))} - e^{-n}}{1-e^{-n}} \right) + (1-\pi)G(a) = G(a) \tag{38}$$

using L'Hopital's rule. Therefore, $\tilde{a}(n) \to E_G(a)$.

Part 3. Taking the limit as $n \to \infty$, we have

$$\lim_{n\to \infty} \tilde{G}_\pi(a;n) = \pi \lim_{n\to \infty} \left(\frac{e^{-n(1-G(a))} - e^{-n}}{1-e^{-n}} \right) + (1-\pi)G(a) = (1-\pi)G(a) \tag{39}$$

for any $a \in [a_0, \bar{a})$ and $\lim_{n\to\infty} \tilde{G}_\pi(\bar{a}; n) = 1$. Therefore, $\tilde{a}(n) \to \pi\bar{a} + (1-\pi)E_G(a)$.

Part 4. For $n > 0$, we have $\tilde{G}_\pi(a; n) < G(a)$ for $a \in A$. To see this, let $w_k(n) = P_k(n)/\alpha(n)$. Using (36), $\tilde{G}_\pi(a; n) = \sum_{k=1}^{\infty} w_k(n)[\pi(G(a))^k + (1-\pi)G(a)]$. Since $\tilde{G}_\pi(a; n)$ is a weighted average of the term $\pi(G(a))^k + (1-\pi)G(a)$ for all $k > 1$, and $(G(a))^k < G(a)$ for all $k > 1$ and $a \in (a_0, \bar{a})$, and $G(a)^k = G(a)$ for $a = a_0$ or $a = \bar{a}$, we have $\tilde{G}_\pi(a; n) < G(a)$ for all $\pi \in (0, 1]$. Therefore, $\tilde{G}_\pi(a; n)$ first order stochastically dominates $G(a)$ and $\tilde{a}(n) > E_G(a)$.

Part 5. Let $f : A \to \mathbb{R}_+$ such that $f' > 0$. For any n_1 and n_2 such that $n_1 > n_2$, Part 6 implies $\tilde{f}(n_1) > \tilde{f}(n_2)$, i.e. $\int_{a_0}^{\bar{a}} f(a) d\tilde{G}(a; n_1) > \int_{a_0}^{\bar{a}} f(a) d\tilde{G}(a; n_2)$. Thus $\tilde{G}_\pi(a; n_1) \leq \tilde{G}_\pi(a; n_2)$ and $\tilde{G}_\pi(a; n_1)$ first order stochastically dominates $\tilde{G}_\pi(a; n_2)$.

Part 6. Applying Leibniz' integral rule gives us

$$\tilde{f}'(n) = \int_{a_0}^{\bar{a}} f(a) \frac{\partial \tilde{g}_\pi(a; n)}{\partial n} da. \tag{40}$$

First, we show that there exists a unique cutoff $\hat{a} \in A$ such that $\frac{\partial \tilde{g}_\pi(a;n)}{\partial n} > 0$ for $a > \hat{a}$ and $\frac{\partial \tilde{g}_\pi(a;n)}{\partial n} < 0$ for $a < \hat{a}$. To start with, we have

$$\tilde{g}_\pi(a; n) = \pi \left(\frac{n g(a) e^{-n(1-G(a))}}{1 - e^{-n}} \right) + (1-\pi) g(a). \tag{41}$$

Differentiating (41) with respect to n, we obtain

$$\frac{\partial \tilde{g}_\pi(a; n)}{\partial n} = \pi g(a) \left[\frac{e^{-n(1-G(a))}[(1 - n(1-G(a)))(1-e^{-n}) - ne^{-n}]}{(1-e^{-n})^2} \right] \tag{42}$$

and therefore $\frac{\partial \tilde{g}_\pi(a;n)}{\partial n} > 0$ if and only if

$$(1 - n(1 - G(a)))(1 - e^{-n}) - ne^{-n} > 0, \tag{43}$$

or, equivalently,

$$G(a) > \frac{1}{1 - e^{-n}} - \frac{1}{n}. \tag{44}$$

Defining $\hat{a} = G^{-1}\left(\frac{1}{1-e^{-n}} - \frac{1}{n}\right)$, we have $\frac{\partial \tilde{g}_\pi(a;n)}{\partial n} > 0$ if and only if $a > \hat{a}$.

We can use the cutoff $\hat{a}$ to rewrite $\tilde{f}'(n)$ as follows:

$$\tilde{f}'(n) \equiv \int_{a_0}^{\hat{a}} f(a)\frac{\partial \tilde{g}_\pi(a;n)}{\partial n}da + \int_{\hat{a}}^{\bar{a}} f(a)\frac{\partial \tilde{g}_\pi(a;n)}{\partial n}da. \tag{45}$$

We therefore have $\tilde{f}'(n) > 0$ if and only if

$$\int_{\hat{a}}^{\bar{a}} f(a)\frac{\partial \tilde{g}_\pi(a;n)}{\partial n}da > -\int_{a_0}^{\hat{a}} f(a)\frac{\partial \tilde{g}_\pi(a;n)}{\partial n}da > 0. \tag{46}$$

Given that $f'(a) > 0$, and both sides of (46) are positive, by definition of $\hat{a}$, a sufficient condition for $\tilde{f}'(n) > 0$ is

$$\int_{\hat{a}}^{\bar{a}} f(\hat{a})\frac{\partial \tilde{g}_\pi(a;n)}{\partial n}da \geq -\int_{a_0}^{\hat{a}} f(\hat{a})\frac{\partial \tilde{g}_\pi(a;n)}{\partial n}da, \tag{47}$$

which is true iff $\int_{\hat{a}}^{\bar{a}} \frac{\partial \tilde{g}_\pi(a;n)}{\partial n}da \geq -\int_{a_0}^{\hat{a}} \frac{\partial \tilde{g}_\pi(a;n)}{\partial n}da$, or equivalently $\int_{a_0}^{\bar{a}} \frac{\partial \tilde{g}_\pi(a;n)}{\partial n}da \geq 0$. Applying Leibniz' integral rule again, $\int_{a_0}^{\bar{a}} \frac{\partial \tilde{g}_\pi(a;n)}{\partial n}da = \frac{\partial}{\partial n}\int_{a_0}^{\bar{a}} \tilde{g}_\pi(a;n)da = 0$, since $\int_{a_0}^{\bar{a}} \tilde{g}_\pi(a;n)da = 1$. Therefore, we have $\tilde{f}'(n) > 0$. ■

Proof of Proposition 1

Taking $\pi \in (0, 1]$ as given, the first-order condition with respect to q_a is

$$\alpha(n)[au'(q_a) - c'(q_a)]\tilde{g}_\pi(a;n) = 0 \tag{48}$$

and the first order-condition with respect to n is

$$\alpha'(n)\tilde{s}(n;\{q_a\}_{a\in A}) + \alpha(n)\tilde{s}'(n;\{q_a\}_{a\in A}) = K. \tag{49}$$

We can verify that $s^*_a = au(q^*_a) - c(q^*_a)$ is strictly increasing in a. Differentiating,

$$\frac{ds^*_a}{da} = u(q^*_a) + [au'(q^*_a) - c'(q^*_a)]\,\frac{dq^*}{da}. \tag{50}$$

Since $au'(q^*_a) - c'(q^*_a) = 0$ by (48) if $n^* > 0$, we have $\frac{ds^*_a}{da} = u(q^*_a) > 0$ for all $a \in (a_0, \bar{a}]$. Given that s^*_a is strictly increasing in a and $s^*_0 \geq 0$, where $s^*_0 \equiv a_0u(q_0) - c(q_0)$ and $q_0 = q(a_0)$, we have $s^*_a \geq 0$ for all $a \in A$. Therefore, all chosen goods $a \in A$ are traded if $a_0 > 0$, and q_a satisfies $au'(q_a) = c'(q_a)$. If $a_0 = 0$, we have $q_a = 0$ since

$\lim_{q\to 0} c'(q)/u'(q) = 0$.

Since s_a^* is strictly increasing in a, the planner chooses the seller with the highest utility shock a whenever possible, i.e. with probability π, and randomizes across sellers otherwise, i.e. with probability $1-\pi$. The distribution of chosen goods, $\tilde{G}_\pi(a;n)$, is therefore equal to (5).

Existence and uniqueness of the solution to the planner's problem follows from Proposition 2. For the planner's problem, we know that $s_a^* \geq 0$ for all $a \in A$ and thus all chosen goods are traded. Setting $i = 0$ in Proposition 2 results in equilibrium conditions that are equivalent to the planner's first-order conditions. It follows that there exists a unique solution to the planner's problem with $n^* > 0$ provided that K satisfies Assumption 4, except that $q_a^0 = q_a^*$ here since q_a^* does not depend directly on n. That is, Assumption 2 suffices. ■

Assumption on cost of entry K

Assumption 4 says the expected trade surplus in the limit as $n \to 0$ must be greater than K, otherwise no sellers enter. Since $\tilde{G} \to G$ as $n \to 0$ by Lemma 1, $\lim_{n\to 0} \tilde{s}(n) = E_G[au(q_a^0) - c(q_a^0)]$ where $q_a^0 \equiv \lim_{n\to 0} q_a(n)$ is given by Lemma 2.

Lemma 2. *For all $a \in [a_0, a_b]$, $q_a^0 = 0$ and, for all $a \in (a_b, \bar{a}]$, q_a^0 satisfies*

$$\left(a - \frac{1-G(a)}{g(a)}\right) u'(q_a) = c'(q_a) \tag{51}$$

where $a_b^0 \in [a_0, \bar{a})$ is the unique solution to $\psi_G(a) = 0$.

The proof of Lemma 2 is identical to the proof of Lemma 8 in Mangin (2023).

Proof of Proposition 2

As in Mangin (2023), our strategy is to solve for the equilibrium in two stages. First, we take z and n as given and solve for $\{(q_a, d_a)\}_{a\in A}$ (inner maximization problem). Second, we solve for z and n (outer maximization problem) given the solutions for $\{(q_a, d_a)\}_{a\in A}$. Next, we use the results to prove Parts 1 to 7 of Proposition 2. Except for the proofs of Part 8 and Lemma 3, and the proofs of existence and uniqueness of equilbrium, which are included below, the derivations of the solution to the inner maximization problem and the outer maximization problem are the same as those in Mangin (2023), except that $\tilde{G}$ is replaced by $\tilde{G}_\pi$.

Proof of Lemma 3 for Proposition 2

Lemma 3. *For any $\pi \in (0,1]$, the function $q(.)$ is weakly increasing for all $a \in A$ and $q'(a) > 0$ for all $a \in (a_b, a_c)$.*

Proof. For all $a \leq a_b$, we have $q_a = 0$ and $q'(a) = 0$. For all $a \geq a_c$, we have that q_a is constant and thus $q'(a) = 0$. For $a \in (a_b, a_c)$, implicit differentiation of

$$(a - \phi(a;n))u'(q_a) = c'(q_a) \tag{52}$$

yields

$$q'(a) = \frac{-[1 - \phi'(a)]u'(q_a)}{[a - \phi(a;n)]u''(q_a) - c''(q_a)} \tag{53}$$

where $\phi(a;n)$ can be simplified to:

$$\phi(a;n) = -\left(\frac{(1-\delta)(1-\tilde{G}_\pi(a;n)) + \frac{i}{\alpha(n)}}{\delta\tilde{g}(a;n)}\right). \tag{54}$$

Differentiating the above yields

$$\phi'(a) = \frac{1-\delta}{\delta} + \frac{\left[(1-\delta)(1-\tilde{G}_\pi(a;n)) + \frac{i}{\alpha(n)}\right]\tilde{g}'_\pi(a;n)}{\delta\tilde{g}(a;n)^2}. \tag{55}$$

Since $u'(q_a) > 0$ and $u''(q_a) < 0$ and $c''(q_a) > 0$ and $a - \phi(a;n) > 0$, we have $q'(a) \geq 0$ provided that $\phi'(a) < 1$. Rearranging, this is true if and only if

$$\left(\frac{(1-\delta)(1-\tilde{G}_\pi(a;n)) + \frac{i}{\alpha(n)}}{\tilde{G}_\pi(a;n)}\right)\left(\frac{\tilde{g}'_\pi(a;n)\tilde{G}_\pi(a;n)}{\tilde{g}_\pi(a;n)^2}\right) < 2\delta - 1. \tag{56}$$

To prove this, we need only consider the case where $\tilde{g}'_\pi(a;n) > 0$ and $\phi(a;n) < 0$. We first show the following:

$$\frac{(1-\delta)(1-\tilde{G}_\pi(a;n)) + \frac{i}{\alpha(n)}}{\tilde{G}_\pi(a;n)} < \delta - \frac{1}{2}. \tag{57}$$

Rearranging the above and simplifying, this is equivalent to

$$\delta + \frac{1}{2}\tilde{G}_\pi(a;n)) > 1 + \frac{i}{\alpha(n)}. \tag{58}$$

For any $a \in (a_b, a_c)$, this is true if $\delta \geq 1 + \frac{i}{\alpha(n)}$, which is true since

$$\delta = \frac{1 - \tilde{G}_\pi(a_b;n) + \frac{i}{\alpha(n)}}{1 - \tilde{G}_\pi(a_b;n) - a_b\tilde{g}_\pi(a_b;n)} \geq 1 + \frac{i}{\alpha(n)(1 - \tilde{G}_\pi(a_b;n))}. \tag{59}$$

We know from the proof of Lemma 17 in Mangin (2023) that

$$\frac{\tilde{g}_1'(a;n)\tilde{G}_1(a;n)}{\tilde{g}_1(a;n)^2} \leq \frac{\mathbb{P}_0''(z)\mathbb{P}_0(z)}{\mathbb{P}_0'(z)^2} \tag{60}$$

for $\pi = 1$ and a general meeting technology $\mathbb{P}_k$ that is invariant. Since the Poisson distribution is invariant, and we have

$$\frac{\mathbb{P}_0''(z)\mathbb{P}_0(z)}{\mathbb{P}_0'(z)^2} = 1 \tag{61}$$

using the fact that $\mathbb{P}_0(z) = e^{-z}$, we obtain

$$\frac{\tilde{g}_1'(a;n)\tilde{G}_1(a;n)}{\tilde{g}_1(a;n)^2} \leq 1. \tag{62}$$

Also, it is clear by assumption $G'' < 0$ that

$$\frac{\tilde{g}_0'(a;n)\tilde{G}_0(a;n)}{\tilde{g}_0(a;n)^2} = \frac{G''(a)G(a)}{g(a)^2} \leq 0 \tag{63}$$

in the limit as $\pi \to 0$. Thus, for $\pi = 1$ and $\pi \to 0$ we have

$$\frac{\tilde{g}_\pi'(a;n)\tilde{G}_\pi(a;n)}{\tilde{g}_\pi(a;n)^2} \leq 1. \tag{64}$$

For any $\pi \in (0,1)$, letting $z = n(1-G(a))$, we can write

$$\tilde{G}_\pi(a;n) = \pi\left(\frac{\alpha'(z)-\alpha'(n)}{\alpha(n)}\right) + (1-\pi)G(a) \tag{65}$$

$$\tilde{g}_\pi(a;n) = \left(\pi\frac{n\alpha'(z)}{\alpha(n)} + 1 - \pi\right) g(a) \tag{66}$$

$$\tilde{g}'_\pi(a;n) = \left(\pi\frac{n\alpha'(z)}{\alpha(n)} + 1 - \pi\right) G''(a) + \pi\frac{n^2\alpha'(z)}{\alpha(n)} g(a)^2 \tag{67}$$

using the fact that $\alpha'(z) = e^{-z}$ and $-\alpha''(z) = \alpha'(z)$ since $\mathbb{P}_k$ is Poisson. Now,

$$\begin{aligned}\frac{\tilde{g}'_\pi(a;n)\tilde{G}_\pi(a;n)}{\tilde{g}_\pi(a;n)^2} &= \frac{\pi\tilde{G}_1(a;n)\left(\left(\pi\frac{n\alpha'(z)}{\alpha(n)} + 1 - \pi\right) G''(a) + \pi\frac{n^2\alpha'(z)}{\alpha(n)} g(a)^2\right)}{\left(\pi\frac{n\alpha'(z)}{\alpha(n)} + 1 - \pi\right)^2 g(a)^2} \\ &\quad + \frac{(1-\pi)G(a)\left(\left(\pi\frac{n\alpha'(z)}{\alpha(n)} + 1 - \pi\right) G''(a) + \pi\frac{n^2\alpha'(z)}{\alpha(n)} g(a)^2\right)}{\left(\pi\frac{n\alpha'(z)}{\alpha(n)} + 1 - \pi\right)^2 g(a)^2}.\end{aligned} \tag{68}$$

Simplifying further, using the fact that $G'' < 0$ by assumption, we have

$$\frac{\tilde{g}'_\pi(a;n)\tilde{G}_\pi(a;n)}{\tilde{g}_\pi(a;n)^2} \leq 1 + \left(\frac{1-\pi}{\pi}\right)\frac{G(a)\alpha(n)}{\alpha'(z)}. \tag{69}$$

Therefore, using the fact that $\tilde{G}_\pi(a;n) \geq \tilde{G}(a)$, we obtain

$$\frac{\tilde{g}'_\pi(a;n)\tilde{G}_\pi(a;n)}{\tilde{g}_\pi(a;n)^2} \leq 2. \tag{70}$$

Therefore, combining (57) and (70), we have proven (56), so we have $q'(a) > 0$ for all $a \in (a_b, a_c)$ for all $\pi \in (0,1]$. ■

Proof of Parts 1 to 8 of Proposition 2

The proof of Parts 1 to 7 is the same as the proof of Parts 1 to 7 in Proposition 6 in Mangin (2023), except that $\tilde{G}$ is replaced by $\tilde{G}_\pi$ for any given $\pi \in (0,1]$.

Part 8. Since v_a is increasing in a, the highest draw is always chosen by buyers whenever possible, i.e. with probability π, and buyers randomize otherwise, i.e. with probability $1-\pi$. Therefore the cdf of chosen goods is given by (5). ■

Proof of existence and uniqueness for Proposition 2

For any $\pi \in (0, 1]$, we first prove existence and uniqueness of the solution to the inner maximization problem and then prove the same for the outer maximization problem. For the inner maximization part, fixing any $\pi \in (0, 1]$, we need to prove that, given z and n from the outer maximization problem, the solution to the inner maximization problem exists and is unique. The proof of the inner maximization part is identical to that found in the proof of Proposition 6 in Mangin (2023) except that $\tilde{G}$ is replaced by $\tilde{G}_\pi$ for any given $\pi \in (0, 1]$, so we omit this proof.

For the outer maximization part, fixing any $\pi \in (0, 1]$, we need to prove that, given $\{(q_a, v_a)\}_{a\in A}$ from the inner maximization problem, the solution (n, z) to the outer maximization problem exists and is unique, and n, z are interior solutions with $n, z > 0$ if Assumption 4 holds. To establish this result, we first prove that, for any $\pi \in (0, 1]$, there exists a non-empty set of solutions n, denoted by $N(K)$, that solves the problem. We then show that equilibrium is unique if $n > 0$ for all $n \in N(K)$, and finally we prove that $n > 0$ for any $n \in N(K)$.

Taking $\pi \in (0, 1]$ as given, and taking $\{(q_a, v_a)\}_{a\in A}$ as given by the inner maximization problem, and ignoring constants, the outer maximization problem is

$$\max_{z,n} \left\{ \alpha(n) \int_{a_0}^{\bar{a}} \left[au(q_a) - \frac{d_a}{\gamma} \right] d\tilde{G}_\pi(a; n) + (\Sigma_{a_c} - i) \frac{z}{\gamma} \right\}, \tag{71}$$

subject to

$$\frac{\alpha(n)}{n} \int_{a_0}^{\bar{a}} \left[-c(q_a) + \frac{d_a}{\gamma} \right] d\tilde{G}_\pi(a; n) \leq K \tag{72}$$

and $n \geq 0$ with complementary slackness, where $\{(q_a, v_a)\}_{a\in A}$ solves the inner maximization problem.

The proof of Lemma 4 is identical to the proof of Lemma 18 in Mangin (2023) except that $\tilde{G}$ is replaced by $\tilde{G}_\pi$ for any given $\pi \in (0, 1]$, so we omit this proof.

Lemma 4. *For any $\pi \in (0, 1]$, the set of solutions $N(K)$ is nonempty and upper hemicontinuous.*

For any $\pi \in (0, 1]$, the following lemma establishes that any strictly positive solution $n \in N(K)$ must be unique. Because $z = d_{a_c} > 0$ where $d_a/\gamma = au(q_a) - v_a$, and $\{(q_a, v_a)\}_{a\in A}$ is given by the inner maximization problem, it follows from Lemma 5 that any solution (n, z) where $n > 0$ is unique.

Lemma 5. *For any* $\pi \in (0, 1]$*, if* $N^+ \subseteq N(K)$ *and* $N^+ \subseteq \mathbb{R}_+\backslash\{0\}$*, then* $N^+ = \{n\}$*.*

Proof. Fix $\pi \in (0, 1]$ and consider any solution $n \in N(K)$ such that $n > 0$. Defining $\Phi(n) \equiv \alpha(n)\tilde{v}(n)$, the solutions n satisfy the first-order condition (27), which is equivalent to

$$\alpha'(n)\tilde{v}(n) + \alpha(n)\tilde{v}'(n) = 0 \tag{73}$$

or $\Phi'(n) = 0$. We show that $\Phi''(n) < 0$ and thus any solution is unique. Using (41), for any $\pi \in (0, 1]$ we have

$$\Phi(n) = \pi \int_{a_0}^{\bar{a}} ne^{-n(1-G(a))} v_a g(a)da + (1 - \pi)(1 - e^{-n}) \int_{a_0}^{\bar{a}} v_a g(a)da. \tag{74}$$

Using Leibniz's integral rule, plus the envelope theorem,

$$\begin{aligned} \Phi'(n) &= \pi \left(\int_{a_0}^{\bar{a}} e^{-n(1-G(a))} v_a g(a)da - \int_{a_0}^{\bar{a}} n(1 - G(a))e^{-n(1-G(a))} v_a g(a)da \right) \\ &\quad +(1 - \pi)e^{-n} \int_{a_0}^{\bar{a}} v_a g(a)da. \end{aligned} \tag{75}$$

By integration by parts on the second integral in $\Phi'(n)$ above, we obtain

$$\Phi'(n) = \pi \left(\int_{a_0}^{\bar{a}} e^{-n(1-G(a))}(1 - G(a))v'(a)da + e^{-n}v(a_0) \right) + (1-\pi)e^{-n} \int_{a_0}^{\bar{a}} v_a g(a)da > 0. \tag{76}$$

Differentiating (76), we find that

$$\Phi''(n) = -\left(\pi \int_{a_0}^{\bar{a}} e^{-n(1-G(a))}(1 - G(a))^2 v'(a)da + \pi e^{-n}v(a_0) + (1 - \pi)e^{-n} \int_{a_0}^{\bar{a}} v_a g(a)da \right) < 0. \tag{77}$$

The fact that $\Phi''(n) < 0$ follows from the fact that $v'(a) = u(q_a) \geq 0$ for all a and $v'(a) > 0$ for some a and also $v(a_0) = 0$. Therefore, any solution $n > 0$ is unique. ■

From Lemma 4, we know that, for any $\pi \in (0, 1]$ and any $K \geq 0$, there exists a non-empty set of solutions $N(K)$ that solves problem (71). We also know that any solution z is interior, since $z/\gamma = \bar{a}u(q_{\bar{a}})$ implies $v_{\bar{a}} = \bar{a}u(q_{\bar{a}}) - \bar{z}/\gamma = 0$ and therefore

$v_a = 0$ for all $a \in A$. We now prove that, for any $n \in N(K)$, we have $n \in \mathbb{R}_+ \backslash \{0\}$ provided Assumption 4 holds. Also, the function $n(K)$ is strictly decreasing in K.

Lemma 6. *For any $\pi \in (0,1]$, any solution $n \in N(K)$ is interior, i.e. $n \in \mathbb{R}_+ \backslash \{0\}$, and the function $n(K)$ is strictly decreasing in K.*

Proof. Fix $\pi \in (0,1]$. First, we show there exists an interior solution $n > 0$. Define $\Lambda(n) \equiv \alpha(n)\tilde{s}(n)$. The first-order condition (27) says $\Lambda'(n) = K$. We prove there exists $n > 0$ such that $\Lambda'(n) = K$ if Assumption 4 holds. We have $\lim_{n\to\infty} \Lambda'(n) = 0$, and

$$\lim_{n\to 0} \Lambda'(n) = \int_{a_0}^{\bar{a}} \lim_{n\to 0} s(a; q_a(n)) dG(a) \tag{78}$$

where $\lim_{n\to 0} s(a; q_a(n)) = s(a; \lim_{n\to 0} q_a(n))$. If the following condition holds:

$$E_G[au(q_a^0) - c(q_a^0)] > K \tag{79}$$

where $q_a^0 \equiv \lim_{n\to 0} q_a(n)$, there exists $n > 0$ that satisfies $\Lambda'(n) = K$ provided that $\Lambda''(n) < 0$ (which we prove below).

Next, any interior solution $n > 0$ is better than $n = 0$. Define the value function:

$$V(K,\gamma) \equiv \max_{z,n} \left\{ \alpha(n) \int_{a_0}^{\bar{a}} \left[au(q_a) - \frac{d_a}{\gamma} \right] d\tilde{G}_\pi(a;n) + (\Sigma_{a_c} - i)\, \frac{z}{\gamma} \right\}. \tag{80}$$

Since we know that z is interior, we have $V(K,\gamma) \equiv \max_n \{\alpha(n)\tilde{v}(n)\}$ since $\int_{a_0}^{\bar{a}} \mu_a = i$. If $n = 0$ then $V(K,\gamma) = 0$. If $n > 0$, $V(K,\gamma) \equiv \max_n \{\alpha(n)\tilde{s}(n) - nk\}$ using constraint (72) with equality. Letting $\Lambda(n) = \alpha(n)\tilde{s}(n)$, we have $V(K,\gamma) > 0$ if $\Lambda(n) - nk > 0$. Thus the candidate solution $n > 0$ is better than $n = 0$ if $\Lambda(n) > nk$ for $n > 0$. Using the fact that $\Lambda'(n) = K$, it suffices to show that $\Lambda''(n) < 0$ and $\frac{\Lambda'(n)n}{\Lambda(n)} < 1$ for $n > 0$. Similarly to Lemma 5, using (41), for any $\pi \in (0,1]$ we have

$$\Lambda(n) = \pi \int_{a_0}^{\bar{a}} ne^{-n(1-G(a))} s(a)g(a)da + (1-\pi)(1-e^{-n}) \int_{a_0}^{\bar{a}} s(a)g(a)da \tag{81}$$

and using Leibniz's integral rule, plus the envelope theorem, yields

$$\Lambda'(n) = \pi\left(\int_{a_0}^{\bar a} e^{-n(1-G(a))}s_a g(a)da - \int_{a_0}^{\bar a} n(1-G(a))e^{-n(1-G(a))}s_a g(a)da\right)$$
$$+(1-\pi)e^{-n}\int_{a_0}^{\bar a} s_a g(a)da. \tag{82}$$

Therefore, we have

$$\frac{\Lambda'(n)n}{\Lambda(n)} = \frac{\pi\int_{a_0}^{\bar a} ne^{-n(1-G(a))}s_a g(a)da + (1-\pi)ne^{-n}\int_{a_0}^{\bar a} s_a g(a)da}{\pi\int_{a_0}^{\bar a} ne^{-n(1-G(a))}s_a g(a)da + (1-\pi)(1-e^{-n})\int_{a_0}^{\bar a} s_a g(a)da}$$
$$-\frac{\pi\int_{a_0}^{\bar a} n^2(1-G(a))e^{-n(1-G(a))}s_a g(a)da}{\pi\int_{a_0}^{\bar a} ne^{-n(1-G(a))}s_a g(a)da + (1-\pi)(1-e^{-n})\int_{a_0}^{\bar a} s_a g(a)da}. \tag{83}$$

So, $\frac{\Lambda'(n)n}{\Lambda(n)} < 1$ for $n > 0$ provided that $ne^{-n} \leq 1 - e^{-n}$, which is true.

Finally, $\Phi(n) = \Lambda(n) - nk$ for $n > 0$, so $\Phi'(n) = \Lambda'(n) - K$ and $\Phi''(n) = \Lambda''(n)$. Since $\Phi''(n) < 0$ from the proof of Lemma 5, we have $\Lambda''(n) < 0$. It follows that, for any $n \in N(K)$, we have $n > 0$. Since we assume $K > 0$, this implies $n \in \mathbb{R}_+\backslash\{0\}$.

Since n is unique by Lemma 5, there is a function $n : \mathbb{R}_+\backslash\{0\} \to \mathbb{R}_+\backslash\{0\}$ such that $n(K)$ solves $\Lambda'(n) = K$. Clearly, n is decreasing in K since $\Lambda''(n) < 0$. ■