When is competition price-increasing?
The impact of expected competition on prices*

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Abstract

This paper examines the effect of expected competition on markups in a random utility model where the number of firms competing for any given consumer is ex ante uncertain. Firms set prices using “limit pricing” (or Bertrand competition). We derive a simple expression for the expected markup and examine how it varies with the expected number of firms. Greater competition can be either price-increasing or price-decreasing depending on the local curvature of the consumer’s expected utility as a function of the expected number of firms. As a result, the expected markup can vary non-monotonically with the degree of expected competition.

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1 Introduction

What is the effect of competition on prices? Economists traditionally believe that greater competition tends to reduce prices, but theory suggests that this is not always the case. In early papers by Satterthwaite (1979) and Rosenthal (1980), for example, an increase in the number of firms can sometimes lead to an increase in the equilibrium price. More recently, Chen and Riordan (2008) shows that greater competition can be price-increasing, i.e. the duopoly price can sometimes exceed the monopoly price, while Chen and Savage (2011) provides empirical support for this surprising theoretical prediction.

This paper studies the impact of competition on prices in a random utility model where consumer choice is determined by firm-specific i.i.d. utility shocks (Anderson, de Palma, and Thisse, 1992). Gabaix, Laibson, Li, Li, Resnick, and de Vries (2016) considers a broad class of symmetric random utility models (e.g. Perloff and Salop, 1985; Sattinger, 1984; and Hart, 1985) and use extreme value theory to examine the impact of competition on prices when the number of firms becomes large. The authors prove that, in the limit as the number of firms becomes infinite, the elasticity of markups with respect to the number of firms converges to the tail index $\gamma$ of the distribution of utility shocks (a measure of tail fatness). If the distribution has positive tail index $\gamma > 0$, competition is asymptotically price-increasing.\(^1\)

While the asymptotic results presented in Gabaix et al. (2016) are insightful and elegant, they are only a useful approximation for markets where a large number of firms compete for any given consumer. Our paper considers the behavior of markups in an alternative environment where the number of firms competing for a particular consumer is finite but ex ante uncertain from the consumer’s perspective. We consider the impact of expected competition on prices. That is, we examine how expected markups vary with the expected number of firms. There are two different reasons for this approach.

First, there are many real-world environments in which consumers may be ex ante uncertain about the exact number of competing firms. For example, suppose a consumer searches for a product or service on the internet. There may be a large number of firms that could potentially compete for that particular consumer, but there are also various frictions (e.g. regarding a firm’s ability to serve the consumer, or regarding the consumer’s ability to find

\(^1\)If the distribution has negative tail index $\gamma < 0$, on the other hand, competition is asymptotically price-decreasing. If $\gamma = 0$, markups are relatively insensitive to changes in the degree of competition as the number of firms becomes large. See Gabaix et al. (2016) for details.
a firm) that may influence the degree of realized competition for that consumer. These frictions may result in uncertainty regarding how many firms will actually compete for any given consumer. When calculating the ex ante *expected markup* faced by a consumer, it therefore makes sense to take into consideration the uncertainty experienced by the consumer regarding both the random utility shocks and the number of competing firms.

Second—and somewhat surprisingly—this approach yields significantly greater tractability compared to an environment with a fixed number of firms. This enables us to derive a remarkably simple expression for the *expected markup* $\mu(\theta)$ as a function of the *expected number of firms*, $\theta$. In turn, this tractability delivers simple proofs of some general results regarding how the expected markup varies with the expected number of firms.

In our environment, firms sets prices after observing the realizations of both the random number of competing firms and buyers’ random utility shocks. Consumers choose whether or not to purchase a single unit of a good, and which good to purchase, after observing both prices and utility shocks. In equilibrium, firms set prices using *limit pricing* (sometimes referred to as “Bertrand competition”).\(^2\) When there are at least two firms, the equilibrium markup (i.e. price minus marginal cost, $p - c$) is equal to the difference between the highest and the second-highest firm-specific utility shock (or valuation). When there is exactly one firm, the markup is equal to the difference between that firm’s utility shock and the consumer’s outside option (which may be zero).

There are four main reasons why we choose to focus on this type of pricing.

First, Gabaix et al. (2016) show that the equilibrium markups for all of the random utility models they consider are asymptotically proportional to the limit pricing markup, revealing a common “limit pricing” logic underlying this class of models. For example, in the Perloff and Salop (1985) model, which is similar to our environment except that firms set prices before observing the realizations of consumers’ utility shocks, the equilibrium markup is asymptotically proportional to the limit pricing markup.

Second, we find that limit pricing delivers the *efficient* level of entry of firms in our environment, i.e. the level of entry that maximizes the total social surplus minus entry costs. This is because limit pricing ensures that firms are paid their marginal contribution to the social surplus, i.e. the difference between the highest and the second-highest utility

\(^2\)Whether the term “limit pricing” is correct here is controversial. We bypass this debate and simply follow Gabaix et al. (2016) in using this phrase to describe this type of pricing.
shock (or consumer’s outside option, if there is only one firm).

Third, all of the key results in this paper regarding markups can be applied to auctions. This is because the expected limit pricing markup is identical to the winning buyer’s expected surplus in a second-price auction with a random number of bidders.

Finally, this type of pricing is widely used in the literature on trade and macroeconomics. For example, Bernard, Eaton, Jensen, and Kortum (2003) incorporate a variant of this form of pricing into a model of international trade with imperfect competition and heterogeneous firms; a large literature has followed.

This paper derives a simple expression that relates the expected markup \( \mu(\theta) \) to the consumer’s expected utility, \( M(\theta) \), and the expected demand, \( D(\theta) \). The consumer’s expected utility \( M(\theta) \) is the expected utility a consumer receives from either purchasing a good or their outside option. The function \( M(.) \) is not a standard utility function but instead represents the consumer’s expected utility as a function of the expected number of firms. The function \( M(.) \) also depends on the distribution of utility shocks and the value of consumers’ outside option. The expected demand \( D(\theta) \) is the probability that a given firm successfully sells their good as a function of the expected number of firms, \( \theta \).

Our expression for the expected markup is remarkably simple: \( \mu(\theta) = M'(\theta) / D(\theta) \). That is, the expected value of the difference between the highest and second-highest utility shock is equal to the derivative of \( M(\theta) \) divided by expected demand \( D(\theta) \). This connection between the expected markup and expected utility delivers a simple, general condition under which competition is either price-increasing \( (\mu'(\theta) > 0) \) or price-decreasing \( (\mu'(\theta) < 0) \).

In particular, whether or not competition is price-increasing depends on the local curvature of the expected utility function, \( M(.) \). The measure of local curvature is \( r_M(\theta) = -M''(\theta)\theta / M'(\theta) \), which is (at least mathematically) equal to the coefficient of relative risk aversion of the consumer’s expected utility function \( M(.) \). Competition is price-increasing if and only if the local curvature \( r_M(\theta) \) is strictly less than \( \varepsilon_D(\theta) \), the elasticity of expected demand with respect to \( \theta \). Intuitively, as the number of firms rises, the marginal increase \( M'(\theta) \) in the consumer’s expected utility decreases since \( M(.) \) is concave and \( M''(\theta) < 0 \), but if the rate of decrease in \( M'(\theta) \) is sufficiently low, i.e. if \( M(.) \) is not too concave relative to the demand elasticity, then competition is price-increasing and \( \mu'(\theta) > 0 \). Importantly, this is a local condition. Whether or not it holds depends not only on the properties of the distribution of shocks and the value of the consumer’s outside option, but also on the value
of \( \theta \). As a result, markups can vary non-monotonically with the expected number of firms, depending on both the distribution and the consumer’s outside option.

We also derive a simple expression for the consumer surplus and show that it is always strictly increasing in the expected number of competing firms. This is true even when competition is price-increasing, i.e. \( \mu'(\theta) > 0 \). Intuitively, this is because the benefit consumers gain from greater competition due to their higher expected utility more than offsets the higher prices paid for purchasing goods. In this way, consumers are always better off when there are more firms, despite the fact that prices may sometimes be higher.

Outline. Section 2 discusses the related literature and this paper’s contribution. Section 3 presents the model. Section 4 derives the main results of the paper. Section 5 contains asymptotic results. Section 6 presents some examples. Section 7 concludes.

2 Related literature

This paper builds on an existing literature that considers the possibility of price-increasing competition, starting with the classic early papers of Satterthwaite (1979) and Rosenthal (1980), both of which describe environments in which an increase in the number of firms can lead to an increase in prices. In Satterthwaite (1979), the environment features imperfect consumer information, while in Rosenthal (1989) the result is obtained using mixed-strategy pricing. More recently, Chen and Riordan (2008) shows that, in an environment featuring perfect information and pure strategies, the symmetric duopoly price is higher than the single-product monopoly price when consumers’ utility shocks are independent and the distribution has a decreasing hazard rate. While our approach is different, our results are complementary to those found in Chen and Riordan (2008). We consider an environment where the number of firms is ex ante uncertain and we provide a necessary and sufficient condition for greater expected competition to be price-increasing. This condition can be applied to any arbitrary finite expected number of firms.

This paper is also complementary to Chen and Riordan (2007), which considers a different environment. The authors prove that an increase in the number of firms can lead to higher equilibrium prices in the spokes model of nonlocalised spatial competition. Part of the reason why competition may be price-increasing in Chen and Riordan (2007) is that firms can sell
to consumers in different submarkets – some of which are (effectively) duopolistic and some
of which are (effectively) monopolistic. A higher number of firms increases the proportion
of submarkets that are duopolistic, which can affect the overall demand elasticity. As a
result, the equilibrium price can be higher under certain conditions. In our environment,
firms sell in a single market where they may be either one firm, two firms, or more than two
firms competing for the consumer. In our setting, however, this is because the number of
competing firms is random. This assumption enables us to provide a simple, general result
that relates the impact of competition on prices to the local curvature of the consumer’s
expected utility, $M(\theta)$, as a function of the expected number of firms.

As discussed, this paper is most closely related to Gabaix, Laibson, Li, Li, Resnick, and
de Vries (2016). Our approach is complementary because in our environment the number
of competing firms is finite but random. This approach enables us to derive simple, new
expressions and general results regarding the expected markup, the markup elasticity, and
the consumer surplus in terms of the consumer’s expected utility, $M(\theta)$. While Gabaix et al.
(2016) shows that the tail index $\gamma$ of the distribution of utility shocks is key to understanding
the impact of competition on prices, we prove that it is the local curvature of the consumer’s
expected utility, as measured by $r_M(\theta) = -M''(\theta)\theta / M'(\theta)$, that is key in our setting. In
Section 5, we demonstrate that our results are consistent with those of Gabaix et al. (2016)
by showing that this measure of concavity converges to $1 - \gamma$ in the limit as the expected
number of firms becomes large. In this way, the asymptotic results for the limit pricing
markup in Gabaix et al. (2016) can be recovered directly from our general expressions.

This paper is also related to Weyl and Fabinger (2013) and Quint (2014). Both papers
focus on different questions, but point out that the comparative statics of pricing behavior
depends on log-concavity of the demand function. In particular, Weyl and Fabinger (2013)
highlights the fact that competition increases (decreases) markups if the density of the
distribution of utility shocks is log-convex (log-concave). Our results are different for three
main reasons. First, our environment features ex ante uncertainty regarding the number of
competing firms. Second, our criterion for markups to be increasing (decreasing) is local,
not global (such as log-concavity of the density). Third, our criterion for markups to be

\footnote{In an earlier paper, Anderson, De Palma, and Nesterov (1995) consider the Perloff-Salop model and
show that a sufficient condition for markups to be weakly decreasing in the number of firms is log-concavity
of the density of the distribution of utility shocks.}
increasing (decreasing) depends not only on properties of the distribution of utility shocks, but also on the expected number of firms and the value of the consumer’s outside option. Intuitively, this is because it depends on the curvature of the function $M(\cdot)$ that gives consumer’s expected utility, which depends on both the expected number of firms and the consumer’s outside option, as well as the distribution of utility shocks.

To highlight how our results differ from the existing literature, consider the example of a Pareto distribution of utility shocks (which has a log-convex density). Our results imply that competition is not always price-increasing. This differs from existing results in Weyl and Fabinger (2013) and Quint (2014) that imply competition is price-increasing when the distribution of utility shocks is log-convex. In our environment, when the distribution is Pareto, markups vary non-monotonically with the expected number of firms whenever the consumer’s outside option is strictly less than the minimum utility shock. At first, when the expected number of firms is relatively low, competition is price-decreasing. Later, when the expected number of firms is sufficiently high, competition is price-increasing.

Finally, the combination of limit pricing (or second-price auctions) and uncertainty regarding the number of competing firms imply that some of the techniques used in this paper are closely related to those found in the competing auctions literature. In competing auctions, a large number of sellers compete to attract buyers by posting auctions with reserve prices (Peters and Severinov, 1997). Our result that limit pricing delivers an efficient level of firm entry mirrors the well-known result that buyer entry is constrained efficient in competing auctions environments where sellers’ reserve prices are equal to their own valuations (Peters and Severinov, 1997; Albrecht et al., 2012). In our environment, the roles of buyers (consumers) and sellers (firms) are reversed and we consider only a single representative consumer. Our focus is on the effect of greater competition (more firms) on prices.

3 Model

Consider a single product market with a single consumer. The number of firms $n \in \mathbb{N}$ is stochastic. Specifically, the distribution of the number of firms is Poisson, i.e. the probability that there are $n$ firms competing for the consumer is $P_n(\theta) = \frac{\theta^n e^{-\theta}}{n!}$ for all $n \in \mathbb{N}$. The

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expected number of competing firms is $\theta$, i.e. $E(n) = \theta$. We take $\theta$ to be exogenous in order to focus our attention on the effect on markups of varying $\theta$.\footnote{It would be straightforward to endogenize the expected number of firms $\theta$ (as in Section 4.4). The Poisson distribution could also be endogenized, e.g. by considering mixed strategies of potential entrants and then taking the limit as the number of potential entrants becomes large.}

Firms are ex ante identical. Each firm can produce one unit of the good at marginal cost $c \geq 0$. After the number of competing firms $n$ is realized, the consumer draws an i.i.d. valuation $x_i$ for each firm $i \in \{1, 2, ..., n\}$ from an exogenous distribution with cdf $G(.)$.

The following assumption is maintained throughout the paper.

**Assumption 1.** The distribution of utility shocks has a continuous, twice differentiable cdf $G(.)$ with pdf $g = G'$, support $[x_0, \infty)$ where $\infty \in \mathbb{R}_+ \cup \{+\infty\}$, and a finite mean.

To keep the environment as general as possible, we allow the possibility that the consumer can obtain the good at cost $c$ and receive utility $z \in [0, x_0]$. The parameter $z$ represents the consumer’s outside option, which may be zero. We assume the consumer always chooses to purchase the good when indifferent.

Firms set prices simultaneously, after observing both the consumer’s firm-specific utility shocks $x_i$ for each $i \in \{1, 2, ..., n\}$ and the number of competing firms. After observing prices, the consumer decides whether to purchases one unit of the good and, if he chooses to purchase, selects the firm $i$ that maximizes his net utility, $x_i - p_i$. Profits for the successful firm $i$ are given by the markup, which is defined as $\mu_i \equiv p_i - c$.

The timing of events can be summarized as follows.

*Timing of events:*

1. Number of competing firms $n$ is realized
2. Random utility shocks $x_i$ are realized
3. Firms observe shocks
4. Firms set prices simultaneously
5. Consumer observes prices
6. Consumer makes purchase decision
7. Production takes place
8. Firm profits are realized
4 Results

In this section, we present our main results regarding the behavior of equilibrium markups. First, we derive the equilibrium expected markup. Next, we derive a simple expression that relates the expected markup to the consumer’s expected utility and the expected demand. We then derive a simple, general condition under which competition is price-increasing. Finally, we derive a simple expression for the consumer surplus and show that it is always increasing in the expected number of competing firms.

4.1 Equilibrium markup

Suppose that the realizations of both the number of competing firms $n$ and the utility shocks (valuations) are known by all agents. Define $M_n \equiv \max\{x_1, ..., x_n\}$, the highest valuation, and let $S_n$ denote the second-highest valuation.

When there are $n \geq 2$ firms, the firm with the highest valuation sets a price equal to $p = M_n - S_n + c$, which gives the consumer net utility $M_n - p = S_n - c$. That is, the firm sets a price which is just low enough to keep the second-best firm out of competition for the consumer. In equilibrium, the consumer chooses to purchase the good (since $S_n \geq x_0 \geq z$), and he purchases from the firm with the highest valuation. The equilibrium markup is equal to the difference between the highest and second-highest valuations, $\mu = M_n - S_n$.

When there is exactly one firm, the firm sets a price $p = x_1 - z + c$, which gives the consumer net utility $x_1 - p = z - c$. That is, the firm sets a price that ensures the consumer is indifferent between purchasing the good and receiving their outside option. In equilibrium, the consumer purchases from the firm and the markup is $\mu = x_1 - z$.

Without loss of generality, we normalize $c = 0$ throughout the remainder of the paper.

Expected markup. We now show how to calculate the ex ante expected markup. Given a fixed number $n \geq 2$ of firms, it is well known that the expected value of the difference between the first and second order statistic is

\[
E(M_n - S_n) = E_{H_n} \left( \frac{1 - G(x)}{g(x)} \right),
\]

where the expected value is taken with regard to the distribution of the first order statistic,
\( H_n(x) = (G(x))^n \). Therefore, when there are at least two firms, the expected markup is

\[
\mu(n) = \int_{x_0}^{x} nG(x)^{n-1}(1 - G(x))dx.
\]

When there is exactly one firm, the expected markup is

\[
\mu(1) = E_G(x) - z.
\]

If there are no firms, we assume \( \mu(0) = 0 \). This assumption is not important since our object of interest is the expected markup conditional on \( n \geq 1 \), which does not depend on \( \mu(0) \).

In our environment, where the number of competing firms \( n \) is stochastic and \( n \sim P_n(\theta) \), we obtain the following expression for \( \mu(\theta) \), the expected markup (conditional on \( n \geq 1 \)), and the markup elasticity, defined by \( \varepsilon_{\mu}(\theta) \equiv \mu'(\theta)\theta/\mu(\theta) \).

**Proposition 1.** If \( G \) satisfies Assumption 1, the equilibrium expected markup is

\[
\mu(\theta) = \frac{\int_{x_0}^{x} \theta e^{-\theta(1-G(x))}(1 - G(x))dx + \theta e^{-\theta}(x_0 - z)}{1 - e^{-\theta}}.
\]

and the markup elasticity is given by

\[
\varepsilon_{\mu}(\theta) = \frac{\int_{x_0}^{x} \theta e^{-\theta(1-G(x))}(1 - \theta(1 - G(x)))(1 - G(x))dx + \theta e^{-\theta}(x_0 - z)(1 - \theta)}{\int_{x_0}^{x} \theta e^{-\theta(1-G(x))}(1 - G(x))dx + \theta e^{-\theta}(x_0 - z)} - \frac{\theta e^{-\theta}}{1 - e^{-\theta}}.
\]

**Proof.** We start with the following:

\[
\mu(\theta) = \frac{\sum_{n=1}^{\infty} P_n(\theta)\mu(n)}{1 - e^{-\theta}}.
\]

Substituting (2) and (3) into (6), and using the fact that \( P_n(\theta) = \frac{\theta^n e^{-\theta}}{n!} \), we have

\[
\mu(\theta) = \frac{\sum_{n=2}^{\infty} \frac{\theta^n e^{-\theta}}{n!} \int_{x_0}^{x} nG(x)^{n-1}(1 - G(x))dx + \theta e^{-\theta}(E_G(x) - z)}{1 - e^{-\theta}}.
\]

Reversing the integral and the summation on the left-hand side of the numerator in (7),
rearranging, and simplifying yields

\[ \mu(\theta) = \int_{x_0}^{\pi} \theta e^{-\theta} \left( \frac{\theta G(x)}{n!} (1 - G(x)) + \theta e^{-\theta} (E_G(x) - z) \right) \frac{\sum_{n=1}^{\infty} \frac{\theta G(x)}{n!} (1 - G(x)) dx + \theta e^{-\theta} (E_G(x) - z)}{1 - e^{-\theta}} \].

Using the fact that \( \sum_{n=1}^{\infty} \frac{(\theta G(x))^n}{n!} = e^{\theta G(x)} - 1 \), we obtain

\[ \mu(\theta) = \int_{x_0}^{\pi} \theta e^{-\theta} (e^{\theta G(x)} - 1)(1 - G(x)) dx + \theta e^{-\theta} (E_G(x) - z) \frac{1 - e^{-\theta}}{1 - e^{-\theta}} \]

or, equivalently, we have

\[ \mu(\theta) = \int_{x_0}^{\pi} \theta e^{-\theta} (1 - G(x)) dx + \theta e^{-\theta} \left( E_G(x) - \int_{x_0}^{\pi} (1 - G(x)) dx - z \right) \frac{1 - e^{-\theta}}{1 - e^{-\theta}} \].

Given that \( G \) has a finite mean, we have \( \lim_{x \to \pi} x (1 - G(x)) = 0 \) and therefore \( \int_{x_0}^{\pi} x g(x) dx - \int_{x_0}^{\pi} (1 - G(x)) dx = x_0 \) using integration by parts. Letting \( E_G(x) - \int_{x_0}^{\pi} (1 - G(x)) dx = x_0 \) in expression (10), we obtain (4). To derive the markup elasticity (5), use the derivative of (4),

\[ \mu'(\theta) = \frac{\int_{x_0}^{\pi} e^{-\theta(1-G(x))}(1 - \theta(1 - G(x))(1 - G(x)) dx + e^{-\theta}(x_0 - z)(1 - \theta)}{1 - e^{-\theta}} \]

\[ - \frac{e^{-\theta} \left( \int_{x_0}^{\pi} \theta e^{-\theta(1-G(x))} (1 - G(x)) dx + \theta e^{-\theta} (x_0 - z) \right)}{(1 - e^{-\theta})^2} \]

and then substitute into \( \varepsilon_{\mu}(\theta) = \mu'(\theta) / \mu(\theta) \) and simplify.

It is unclear whether the markup elasticity \( \varepsilon_{\mu}(\theta) \) is positive or negative, i.e. whether \( \mu(\theta) \) is increasing or decreasing in the expected number of firms, or degree of competition. That is, by simply examining expression (5), it is not easy to see when exactly competition is price-increasing (\( \mu'(\theta) > 0 \)) or price-decreasing (\( \mu'(\theta) < 0 \)). We will derive a simple expression for the expected markup and the markup elasticity in Sections 4.3 and 4.5. Before presenting these results, we first derive the consumer’s expected utility and some of its properties.
4.2 Consumer’s expected utility

We now derive expressions for the distribution of the consumer’s utility and the expected value of this distribution, which we call the consumer’s expected utility. When there is at least one firm, the consumer’s utility equals the highest utility shock $x$ among $n$ draws when $n \sim P_n(\theta)$. When there are no firms, the consumer receives utility $z$, their outside option.

Let $H_n(\cdot)$ be the cdf of the distribution of the maximum of $n$ draws from $G(x)$, i.e. $H_n(x) \equiv (G(x))^n$. The distribution of the consumer’s utility when $n \sim P_n(\theta)$ is denoted by $H(\cdot; \theta)$ and it is given by the following:

\begin{equation}
H(x; \theta) = \begin{cases}
\sum_{n=0}^{\infty} P_n(\theta) (G(x))^n & \text{if } x \in [z, \bar{x}) \\
0 & \text{if } x \in [0, z)
\end{cases}
\end{equation}

Using $P_n(\theta) = \frac{\theta^n e^{-\theta}}{n!}$, it can be shown that\footnote{This uses the fact that $e^{-\theta} \sum_{n=0}^{\infty} \frac{(\theta G(x))^n}{n!} = e^{-\theta} e^{\theta G(x)} = e^{-\theta (1 - G(x))}$.}.

\begin{equation}
H(x; \theta) = \begin{cases}
e^{-\theta (1 - G(x))} & \text{if } x \in [z, \bar{x}) \\
0 & \text{if } x \in [0, z)
\end{cases}
\end{equation}

The distribution $H(\cdot; \theta)$ has support $[x_0, \bar{x}) \cup \{z\}$. It features a mass point at $z$, since with probability $e^{-\theta}$ there are no firms and the consumer’s utility equals their outside option, $z$.

Now let $M(\theta)$ denote the expected utility of the consumer, i.e. $M(\theta) \equiv E_H(x)$. Lemma 1 summarizes some key properties of the function $M(\cdot)$.

**Lemma 1.** If $G$ satisfies Assumption 1, the consumer’s expected utility is

\begin{equation}
M(\theta) = \int_{x_0}^{\bar{x}} \theta e^{-\theta (1 - G(x))} x g(x) dx + e^{-\theta} z
\end{equation}

and the derivative of $M(\cdot)$ is given by

\begin{equation}
M'(\theta) = \int_{x_0}^{\bar{x}} e^{-\theta (1 - G(x))} (1 - G(x)) dx + e^{-\theta} (x_0 - z).
\end{equation}

The function $M(\cdot)$ has the following properties: (i) $M'(\theta) > 0$; (ii) $M''(\theta) < 0$; (iii) $M(0) =$
z; (iv) \( \lim_{\theta \to \infty} M(\theta) = \bar{x} \); (v) \( \lim_{\theta \to \infty} M'(\theta) = 0 \); and (vi) \( \lim_{\theta \to 0} M'(\theta) = E_G(x) - z \).

**Proof.** Using \( M(\theta) \equiv E_H(x) \) and expression (13) for the distribution \( H(x; \theta) \), we obtain (14). Part (i). Applying Leibniz’s integral rule to (14), we have

\[
M'(\theta) = \int_{x_0}^{\bar{x}} xg(x)e^{-\theta(1-G(x))}dx - \int_{x_0}^{\bar{x}} \theta xg(x)e^{-\theta(1-G(x))}(1-G(x))dx.
\]

By integration by parts on the right integral, and using the fact that \( \lim_{x \to \bar{x}} x(1-G(x)) = 0 \),

\[
\int_{x_0}^{\bar{x}} \theta xg(x)e^{-\theta(1-G(x))}(1-G(x))dx = -x_0e^{-\theta} - \int_{x_0}^{\bar{x}} e^{-\theta(1-G(x))}((1-G(x)) - xg(x))dx.
\]

Substituting (17) into (16) yields (15), and clearly \( M'(\theta) > 0 \). Part (ii). Applying Leibniz’ integral rule again, we obtain

\[
M''(\theta) = -
\left( \int_{x_0}^{\bar{x}} e^{-\theta(1-G(x))}(1-G(x))^2dx + e^{-\theta}(x_0 - z) \right) < 0.
\]

Parts (iii) and (v). It is clear that \( M(0) = z \) and \( \lim_{\theta \to \infty} M'(\theta) = 0 \). Part (iv). Consider \( \lim_{\theta \to \infty} M(\theta) \). Letting \( t = 1 - G(x) \), we have \( M(\theta) = \theta \int_0^1 e^{-\theta t}G^{-1}(1-t)dt + e^{-\theta}z \). Defining \( G^{-1}(y) = 0 \) for \( y < 0 \), we have \( G^{-1}(1-t) = 0 \) for \( t > 1 \) so \( M(\theta) = \theta \int_0^\infty e^{-\theta t}G^{-1}(1-t)dt + e^{-\theta}z \). We can now apply the initial value theorem for Laplace transforms, which states that for any piecewise continuous function \( \phi(t) \), \( \lim_{\theta \to \infty} \theta \int_0^\infty e^{-\theta t}\phi(t)dt = \lim_{t \to 0} \phi(t_0) \). So \( \lim_{\theta \to \infty} M(\theta) = \lim_{t_0 \to 0} G^{-1}(1-t_0) + 0 = G^{-1}(1) = \bar{x} \). Part (vi). Using (16), \( \lim_{\theta \to 0} M'(\theta) = \lim_{\theta \to 0} \int_{x_0}^{\bar{x}} xg(x)e^{-\theta(1-G(x))}dx - z = \int_{x_0}^{\bar{x}} xg(x)dx - z = E_G(x) - z \).

### 4.3 Simple expression for expected markup

We now present a simple expression that relates the expected markup \( \mu(\theta) \) to the consumer’s expected utility \( M(\theta) \) and the ex ante expected demand for a single firm’s product. Let \( D(\theta) \) denote the expected demand faced by single firm, i.e. the probability of a sale. Before we present Proposition 2, the following lemma gives us the expected demand \( D(\theta) \).
Lemma 2. If $G$ satisfies Assumption 1, the expected demand is given by

\begin{equation}
D(\theta) = \frac{1 - e^{-\theta}}{\theta}.
\end{equation}

**Proof.** For a fixed number of firms, it is well-known that the expected demand for a single firm’s product in equilibrium is equal to

\begin{equation}
D(n) = \int_{x_0}^{\pi} g(x)(G(x))^{n-1} dx.
\end{equation}

This is equal to the probability that the firm’s utility shock is higher than that of all $n-1$ of the other firms. Since $D(n)n = \int_{x_0}^{\pi} h_n(x) dx = 1$, we have $D(n) = 1/n$. Given that $n \sim P_n(\theta)$, the expected demand is

\begin{equation}
D(\theta) = \sum_{n=1}^{\infty} P_n^f(\theta) D(n)
\end{equation}

where $P_n^f(\theta)$ is the probability there are $n$ firms from the perspective of firms. Therefore, substituting $P_n^f(\theta) = \frac{\theta^{n-1} e^{-\theta}}{(n-1)!}$ and $D(n) = 1/n$ into (21) and simplifying, we obtain

\begin{equation}
D(\theta) = \frac{e^{-\theta}}{\theta} \sum_{n=1}^{\infty} \frac{\theta^n}{n!}.
\end{equation}

Finally, using the fact that \( \sum_{n=1}^{\infty} \frac{\theta^n}{n!} = e^\theta - 1 \), we obtain (19). $\blacksquare$

The next result presents a simple, general expression for the expected markup $\mu(\theta)$ as a function of the expected number of firms. Proposition 2 states that the expected markup $\mu(\theta)$ is equal to marginal contribution $M'(\theta)$ of an additional firm to the consumer’s expected utility, divided by the expected demand $D(\theta)$. Since we have already derived the relevant expressions for $M'(\theta)$ and $D(\theta)$ in Lemmas 1 and 2, it is straightforward to prove this result.

**Proposition 2.** If $G$ satisfies Assumption 1, the expected markup is given by

\begin{equation}
\mu(\theta) = \frac{M'(\theta)}{D(\theta)}
\end{equation}
where $M(\theta)$ is expected utility and $D(\theta)$ is expected demand.

**Proof.** Dividing expression (15) for $M'(\theta)$ from Lemma 1 by expression (19) for $D(\theta)$ from Lemma 2, we obtain expression (4) from Proposition 1. □

Since $\mu(\theta)$ is the expected value of the difference between the highest and the second-highest utility shock, expression (23) also represents the expected surplus for the winner of a second-price auction when the number of bidders $n$ is stochastic and $n \sim P_n(\theta)$.

The remarkably simple expression for the expected markup given by (23) highlights the tractability of our environment. In particular, Proposition 2 cannot be obtained in an identical environment where the number of firms $n$ is fixed. It is the simplicity of expression (23) in Proposition 2 which enables us to derive a simple, general condition under which competition is either price-increasing or price-decreasing.

To better understand the intuition behind the simple expression presented in Proposition 2, we can think about it in terms of the efficiency of firm entry.

### 4.4 Efficient Entry of Firms

Suppose the expected number of firms $\theta$ is not exogenous but is instead determined by a zero profit condition. If firms pay an entry cost $k > 0$, the zero profit condition says $\Pi(\theta) = k$, where $\Pi(\theta)$ is the ex ante expected payoff for an entering firm (excluding entry cost).\(^7\) Lemma 3 provides a simple expression for $\Pi(\theta)$.

**Lemma 3.** The ex ante expected payoff for a firm is equal to

\[
\Pi(\theta) = D(\theta)\mu(\theta)
\]

where $D(\theta)$ is expected demand and $\mu(\theta)$ is the expected markup.\(^8\)

\(^7\)For simplicity, we are considering a single consumer and the expected number of firms $\theta$. However, we could consider an environment with a large number of consumers $L$ and then determine the equilibrium number of entering firms $V$. The expected number of firms per consumer would then be $\theta \equiv V/L$. The equilibrium $\theta^*$ and the planner’s choice $\theta^P$ would be exactly the same.

\(^8\)Note: this does not say $E(D(n))E(\mu(n)) = E(D(n)\mu(n))$ since $D(\theta)$ is the expected demand from a firm’s perspective while $\mu(\theta)$ is the expected markup from the consumer’s perspective (conditional on $n \geq 1$).
Proof. The expected payoff for a firm is given by

\[
\Pi(\theta) = \sum_{n=1}^{\infty} P^f_n(\theta) D(n) \mu(n),
\]

where \( P^f_n(\theta) \) is the probability there are \( n \) firms from the perspective of firms. Substituting in \( P^f_n(\theta) = P_n(\theta)^n \) and \( D(n) = 1/n \), we obtain

\[
\Pi(\theta) = \sum_{n=1}^{\infty} P_n(\theta) \frac{\mu(n)}{\theta}.
\]

Finally, using expression (6) for \( \mu(\theta) \) yields (24).

Now suppose a social planner were to choose the expected number of firms \( \theta \) in order to maximize the expected social surplus per consumer, \( \Omega(\theta) \equiv M(\theta) - c - k\theta \). Combining Lemma 3 and Proposition 2, it is straightforward to show that the level of entry of firms is efficient under limit pricing (i.e. the planner’s choice \( \theta^P \) is the same as the equilibrium \( \theta^* \)).

**Proposition 3.** With free entry of firms, the socially optimal expected number of firms \( \theta^P \) is equal to the equilibrium expected number of firms \( \theta^* \).

**Proof.** Applying Lemma 3, the zero profit condition says \( D(\theta) \mu(\theta) = k \). At the same time, Proposition 2 says \( D(\theta) \mu(\theta) = M'(\theta) \). Therefore, the zero profit condition says \( M'(\theta) = k \). Since \( M''(\theta) < 0 \), \( \lim_{\theta \to 0} M'(\theta) = E_G(x) - z \), and \( \lim_{\theta \to \infty} M'(\theta) = 0 \) by Lemma 1, there exists a unique equilibrium \( \theta^* > 0 \) provided that \( E_G(x) > z + k \).\(^9\) The first-order condition for the planner’s problem says that the planner’s choice \( \theta^P \) satisfies the same equation, \( M'(\theta) = k \). Therefore, \( \theta^* = \theta^P \) and firm entry is efficient under limit pricing.

The fact that limit pricing delivers the efficient level of entry of firms is the key to understanding Proposition 2. This result says that firms’ expected payoff, \( \Pi(\theta) = D(\theta) \mu(\theta) \), is equal to the marginal contribution \( M'(\theta) \) of an additional firm to the consumer’s expected

\(^9\)Anderson et al. (1995) consider the Perloff-Salop model and show that log-concavity is a sufficient condition for the existence of an equilibrium when there is free entry of firms. In our environment, this assumption is not required for determining firm entry since expected profits are always decreasing in the number of firms, i.e. \( \Pi'(\theta) = M''(\theta) < 0 \), even in cases where the expected markup is increasing in \( \theta \).
utility – and therefore to the social surplus. Intuitively, firms are paid their marginal contribution to the social surplus because the expected markup equals the difference between the highest and second-highest utility shock. In this way, the fact that limit pricing delivers efficiency of entry is really the key to the simple expression in Proposition 2.

In the next section, we exploit the simplicity of the expression in Proposition 2 to derive a condition that is both necessary and sufficient for competition to be price-increasing.

4.5 When is competition price-increasing?

We now present some general results regarding whether competition is price-increasing or price-decreasing. We prove that the local curvature of the consumer’s expected utility, $M(\theta)$, as a function of the expected number of firms, is the key to understanding the impact of competition on prices in our setting. As a result, the effect of competition on markups depends not only on the characteristics of the underlying distribution of utility shocks, but also on the expected number of firms $\theta$ and the value of the consumer’s outside option $z$. In Section 5, we show how to recover the asymptotic results of Gabaix et al. (2016) by taking the limit of our expressions as the expected number of firms $\theta \to \infty$.

Proposition 4 presents a simple expression for the markup elasticity $\varepsilon_\mu(\theta)$, which yields a general condition under which competition is price-increasing (or price-decreasing). This result features a measure of the local curvature, or degree of concavity, of the function $M(.)$, which gives the expected utility of the consumer when the expected number of firms is $\theta$. It is essentially the Arrow-Pratt coefficient of relative risk aversion of the function $M(.)$ at $\theta$.\(^\text{10}\)

Proposition 4 also features the demand elasticity, $\varepsilon_D(\theta)$, defined as the elasticity of the “demand” function $D(.)$ with respect to the expected number of firms, $\varepsilon_D(\theta) \equiv -D'(\theta)\theta / D(\theta)$.

Proposition 4. If $G$ satisfies Assumption 1, the markup elasticity is

\begin{equation}
\varepsilon_\mu(\theta) = \varepsilon_D(\theta) - r_M(\theta)
\end{equation}

where $\varepsilon_D(\theta)$ is the demand elasticity and

\begin{equation}
r_M(\theta) \equiv -\frac{M''(\theta)\theta}{M'(\theta)}.
\end{equation}

\(^{10}\)Of course, $M(.)$ is a function of the expected number of firms $\theta$, not a standard utility function.
The expected markup $\mu(\theta)$ is strictly increasing in the expected number of firms, i.e. $\mu'(\theta) > 0$ and competition is price-increasing, if and only if

\[(29) \quad r_M(\theta) < \varepsilon_D(\theta).\]

**Proof.** Starting with (23), we have $\mu(\theta) = M'(\theta)/D(\theta)$. The elasticity of $\mu(\theta)$ equals the elasticity of the numerator $M'(\theta)$ minus the elasticity of the denominator $D(\theta)$. That is,

\[(30) \quad \varepsilon_\mu(\theta) = \frac{M''(\theta)\theta}{M'(\theta)} - \frac{D'(\theta)\theta}{D(\theta)}.\]

Therefore, $\varepsilon_\mu(\theta) = -r_M(\theta) + \varepsilon_D(\theta)$ and we have $\mu'(\theta) > 0$ if and only if $r_M(\theta) < \varepsilon_D(\theta)$. 

As the expected number of firms rises, the expected value of consumer’s utility, $M(\theta)$, increases. However, the marginal increase $M'(\theta)$ in the expected value $M(\theta)$ is decreasing in $\theta$ since $M''(\theta) < 0$ by Lemma 1. If the rate of decrease in $M'(\theta)$ is sufficiently low, i.e. if $M''(\theta)$ is not too negative and $r_M(\theta)$ is not too high relative to the demand elasticity $\varepsilon_D(\theta)$ (i.e. $M(.)$ is not too concave), then greater competition is price-increasing, i.e. $\mu'(\theta) > 0$.

This condition differs from existing results in Weyl and Fabinger (2013) and Quint (2014) that imply competition is price-increasing when the distribution of utility shocks is log-convex. Importantly, our criterion is local, not global. Whether or not condition (29) holds depends crucially on the local curvature of the consumer’s expected utility, $M(\theta)$, as a function of the expected number of firms. In turn, the local curvature of the function $M(.)$ at a particular value of $\theta$ depends not only on the properties of the distribution of utility shocks but also on the value of $\theta$, as well as the value of the consumer’s outside option.

Since it is a local condition, markups can vary non-monotonically with the expected number of firms. In the next section, we will see some examples of how this may occur.

### 4.6 Consumer surplus

We know that competition is price-increasing whenever condition (29) holds, but the effect on consumer surplus is unclear. Define *consumer surplus* as $\Delta(\theta) \equiv M(\theta) - (1 - e^{-\theta})\mu(\theta)$. That is, we measure consumer surplus as the expected utility of a consumer, $M(\theta)$, minus the expected payment by the consumer (i.e. the probability that a consumer purchases the good
from a firm, $1 - e^{-\theta}$, multiplied by $\mu(\theta)$, the expected markup conditional on purchasing the good). It is not immediately clear whether $\Delta'(\theta) > 0$ since we have $M'(\theta) > 0$ but $\mu'(\theta) > 0$ whenever condition (29) holds and the probability $1 - e^{-\theta}$ is always increasing in $\theta$.

Proposition 5 presents a simple expression for the consumer surplus in terms of the function $M(.)$, consumer’s expected utility. Using this expression, it readily follows that the consumer surplus is always strictly increasing in the expected number of firms.

**Proposition 5.** If $G$ satisfies Assumption 1, the consumer surplus is

\[
\Delta(\theta) = M(\theta) - \theta M'(\theta).
\]

The consumer surplus is strictly increasing in the expected number of firms, i.e. $\Delta'(\theta) > 0$.

**Proof.** Starting with the definition of consumer surplus, $\Delta(\theta) = M(\theta) - (1 - e^{-\theta})\mu(\theta)$, we can use the fact that $\mu(\theta) = M'(\theta)/D(\theta)$ from Proposition 2, and the fact that $D(\theta) = (1 - e^{-\theta})/\theta$ from Lemma 2, to obtain (31). Differentiating (31) yields $\Delta'(\theta) = -\theta M''(\theta)$, so we have $\Delta'(\theta) > 0$ since $M''(\theta) < 0$ by Lemma 1.

Intuitively, the consumer surplus is strictly increasing in the expected number of firms because the benefit consumers receive from having higher expected utility when there are more firms more than offsets any possible increase in the expected payment by the consumer, even when $\mu'(\theta) > 0$. This suggests that while greater competition can indeed be price-increasing, consumers are always better off – as measured by the consumer surplus.

## 5 Asymptotic results

Gabaix et al. (2016) show that, in the limit as the number of firms $n \to \infty$, the markup elasticity $\varepsilon(\mu(n)$ converges to the tail index $\gamma_G$ of the distribution of random utility shocks $G$. Competition is therefore either asymptotically price-increasing (i.e. $\mu'(n) \to 0$ as $n \to \infty$) or asymptotically price-decreasing (i.e. $\mu'(n) < 0$ as $n \to \infty$) depending on whether the tail index is greater than or less than zero, i.e. whether the distribution is fat-tailed or not.

In this section, we demonstrate that these asymptotic results can be recovered directly from our general expressions by taking the limit as $\theta \to \infty$. 
We first derive Lemma 4, which is a variant of Theorem 3 of Gabaix et al. (2016) that is adapted to the present environment. First, we define the notion of regular variation.

**Definition 1.** We say that a function \( h : \mathbb{R}_+ \to \mathbb{R} \) is regularly varying at zero with index \( \rho \), and denote this by \( h \in RV_0^\rho \), if and only if \( h \) is strictly positive in a neighborhood of zero and, for all \( \lambda > 0 \), we have

\[
\lim_{t \to 0} \frac{h(\lambda t)}{h(t)} = \lambda^\rho.
\]

Aside from notational differences, Lemma 4 is identical to Theorem 3 of Gabaix et al. (2016), except for the following differences. First, \( H(\cdot; \theta) \) is the distribution of the consumer’s expected utility when \( n \) is stochastic (not fixed), i.e. \( n \sim \mathcal{P}_n(\theta) \). Second, the consumer’s outside option is \( z \). Finally, we restrict attention to the case where \( \zeta(x) \geq 0 \). The proof is somewhat simpler than that found in Gabaix et al. (2016).

**Lemma 4.** Let \( G \) be a differentiable cdf with support \((x_0, \bar{x})\) that is strictly increasing in a left neighborhood of \( \bar{x} \). Let \( \zeta : (x_0, \bar{x}) \cup \{0\} \to \mathbb{R}^+ \) be a function that satisfies \( \zeta(x) \geq 0 \) for all \( x \in (x_0, \bar{x}) \). Suppose that \( \tilde{\zeta}(t) \equiv \zeta(G^{-1}(1 - t)) \in RV_0^\rho \) with \( \rho > -1 \), and \( \int_{x_0}^{\bar{x}} |\zeta(x)g(x)| \, dx < \infty \). Then, as \( \theta \to \infty \), we have

\[
E_H(\zeta(x)) = \int_{x_0}^{\bar{x}} \zeta(x)g(x)\theta e^{-\theta(1-G(x))} \, dx + e^{-\theta} \zeta(z) \sim \zeta \left( G^{-1} \left( 1 - \frac{1}{\theta} \right) \right) \Gamma(\rho + 1)
\]

where \( \Gamma(t) \equiv \int_0^\infty y^{t-1}e^{-y} \, dy \) is the Gamma function.

**Proof.** Consider the integral \( E_H(\zeta(x)) = \int_{x_0}^{\bar{x}} \zeta(x)g(x)\theta e^{-\theta(1-G(x))} \, dx \) where we assume that \( \zeta(x) \geq 0 \) and \( \tilde{\zeta}(t) \equiv \zeta(G^{-1}(1 - t)) \in RV_0^\rho \) with \( \rho > -1 \), and \( \int_{x_0}^{\bar{x}} |\zeta(x)g(x)| \, dx < \infty \). Changing variables by letting \( t = 1 - G(x) \) and rewriting yields

\[
E_H(\zeta(x)) = \int_0^1 \theta e^{-\theta t} \zeta(G^{-1}(1 - t)) \, dt + e^{-\theta} \zeta(z)
\]

Rewriting, this is equivalent to

\[
E_H(\zeta(x)) = \int_0^1 \theta e^{-(\theta - 1)t} \zeta(G^{-1}(1 - t)) e^{-t} \, dt + e^{-\theta} \zeta(z).
\]
Now define \( h(t) \equiv \zeta(G^{-1}(1-t))e^{-t} \) and \( H(t) \equiv \int_0^t h(y)dy \). Letting \( \theta - 1 = \theta' \), we have

\[
E_H(\zeta(x)) = \int_0^1 \theta e^{-\theta' t} h(t)dt = \int_0^1 \theta e^{-\theta' t} d\hat{H}(t) + e^{-\theta} \zeta(z). \tag{36}
\]

Defining \( \hat{h}(t) = h(t) \) for all \( t \in [0,1] \) and \( \hat{h}(t) = 0 \) for all \( t \in (1,\infty) \), and \( \hat{H}(t) \equiv \int_0^t \hat{h}(y)dy \),

\[
E_H(\zeta(x)) = \int_0^\infty \theta e^{-\theta' t} d\hat{H}(t) + e^{-\theta} \zeta(z). \tag{37}
\]

We can apply Karamata’s Tauberian Theorem since \( \hat{H}(t) \) is weakly positive and weakly increasing in \( t \). This theorem says that if \( \hat{H}(t) \in RV_\alpha^0 \) then as \( \theta' \to \infty \) we have

\[
\int_0^\infty e^{-\theta' t} d\hat{H}(t) \sim \hat{H}(1/\theta')\Gamma(\alpha + 1). \tag{38}
\]

Now, since \( \zeta(t) \equiv \zeta(G^{-1}(1-t))e^{-t} \) with \( \rho > -1 \) by assumption, we have \( h(t) \equiv \zeta(G^{-1}(1-t))e^{-t} \) with \( \rho > -1 \) since \( e^{-t} \in RV_0^0 \), and therefore also \( \hat{h}(t) \in RV_\rho^0 \) with \( \rho > -1 \). By Lemma A1.6 of Gabaix et al. (2016), this implies that \( \hat{H}(t) \equiv \int_0^t \hat{h}(y)dy \in RV_{\rho+1}^0 \) and therefore \( \alpha = \rho + 1 \), so we have

\[
\int_0^\infty e^{-\theta' t} d\hat{H}(t) \sim \hat{H}(1/\theta')\Gamma(\rho + 2) \tag{39}
\]

as \( \theta' \to \infty \). By Lemma A1.6 of Gabaix et al. (2016), we also have \( \lim_{x \to 0} \frac{xh(x)}{H(x)} = \rho + 1 \), and thus \( \hat{H}(x) \sim x\hat{h}(x)/(\rho + 1) \) as \( x \to 0 \). Therefore as \( \theta' \to \infty \) we have

\[
\int_0^\infty e^{-\theta' t} d\hat{H}(t) \sim \frac{1}{\theta'} \frac{\hat{h}(1/\theta')\Gamma(\rho + 2)}{\rho + 1}. \tag{40}
\]

Since \( \Gamma(\rho + 2)/(\rho + 1) = \Gamma(\rho + 1) \) and \( \hat{h}(t) = \zeta(G^{-1}(1-t))e^{-t} \) for all \( t \in [0,1] \),

\[
\int_0^\infty e^{-\theta' t} d\hat{H}(t) \sim \frac{1}{\theta'} \zeta \left( G^{-1} \left( 1 - \frac{1}{\theta'} \right) \right) e^{-1/\theta'} \Gamma(\rho + 1). \tag{41}
\]
Finally, using the fact that $\theta' = \theta - 1$, in the limit as $\theta \to \infty$ we have

$$
E_H(\zeta(x)) = \int_0^\infty \theta e^{-\theta t} d\hat{H}(t) + e^{-\theta} \zeta(z) \sim \zeta \left( G^{-1} \left( 1 - \frac{1}{\theta} \right) \right) \Gamma(\rho + 1).
$$

Before we present the next lemma, we provide a preliminary definition.

**Definition 2.** We say that $G$ is well-behaved if and only if it satisfies Assumption 1 and

$$
\lim_{x \to \pi} \frac{1 - G(x)}{g(x)} = a
$$

where $a \in \mathbb{R}^+ \cup \{+\infty\}$

and $G$ has finite tail index $\gamma_G$ given by

$$
\lim_{x \to \pi} \frac{d}{dx} \left( \frac{1 - G(x)}{g(x)} \right) = \gamma_G \quad \text{where } \gamma_G \in \mathbb{R}.
$$

Lemma 5 states that, in the limit as the expected number of firms becomes large, we have $r_M(\theta) \to 1 - \gamma_G$. This result is used here to prove Proposition 6, but it is interesting in its own right because it says that the tail index of the distribution of utility shocks – which is a measure of tail fatness – is equal to one minus the asymptotic value of $r_M(\theta)$ – a measure of local curvature of the consumer’s expected utility function, $M(.)$.

**Lemma 5.** Assume that Lemma 4 applies and $G$ is well-behaved. In the limit as $\theta \to \infty$, we have $r_M(\theta) \to 1 - \gamma_G$, where $\gamma_G$ is the tail index of $G$.

**Proof.** Starting with definition (28) of $r_M(\theta)$, and using (15) and (18),

$$
\lim_{\theta \to \infty} r_M(\theta) = \lim_{\theta \to \infty} \int_{x_0}^\pi \theta e^{-\theta(1-G(x))} (1 - G(x))^2 dx + \theta e^{-\theta} (x_0 - z).
$$

Rearranging (45) and simplifying, this is equivalent to

$$
\lim_{\theta \to \infty} r_M(\theta) = \lim_{\theta \to \infty} \frac{\theta \int_{x_0}^\pi \left( \frac{(1-G(x))^2}{g(x)} \right) g(x) \theta e^{-\theta(1-G(x))} dx}{\int_{x_0}^\pi \left( \frac{1-G(x)}{g(x)} \right) g(x) \theta e^{-\theta(1-G(x))} dx}.
$$
Applying Lemma 4 to the numerator of (46), where \( \zeta(x) = \frac{(1-G(x))^2}{g(x)} \), we have

\[
(47) \quad \theta \int_{x_0}^{\pi} \frac{(1-G(x))^2}{g(x)} g(x) \theta e^{-\theta (1-G(x))} dx \sim_{\theta \to \infty} \frac{\theta (1 - G(G^{-1}(1 - 1/\theta)))^2}{g(G^{-1}(1 - 1/\theta))} \Gamma(\rho_1 + 1).
\]

To determine \( \rho_1 \), let \( \hat{\zeta}(t) \equiv \frac{t^2}{g(G^{-1}(1-t))} \). Using Lemma A1 in the Appendix of Gabaix et al. (2016), we have \( \hat{\zeta}(t) \in RV^0_{\rho_1} \) where \( \rho_1 = 2 - (\gamma_G + 1) = 1 - \gamma_G \). Similarly, applying Lemma 4 to the denominator of (46), where \( \zeta(x) = \frac{1-G(x)}{g(x)} \), we obtain

\[
(48) \quad \int_{x_0}^{\pi} \frac{1-G(x)}{g(x)} g(x) \theta e^{-\theta (1-G(x))} dx \sim_{\theta \to \infty} \frac{(1-G(G^{-1}(1 - 1/\theta)))}{g(G^{-1}(1 - 1/\theta))} \Gamma(\rho_2 + 1).
\]

To determine \( \rho_2 \), let \( \hat{\zeta}(t) = \frac{t}{g(G^{-1}(1-t))} \). Using Lemma A1 in the Appendix of Gabaix et al. (2016), we have \( \hat{\zeta}(t) \in RV^0_{\rho_2} \) where \( \rho_2 = 1 - (\gamma_G + 1) = -\gamma_G \). Therefore,

\[
(49) \quad \lim_{\theta \to \infty} r_M(\theta) = \lim_{\theta \to \infty} \theta (1 - G(G^{-1}(1 - 1/\theta))) \frac{\Gamma(2 - \gamma_G)}{\Gamma(1 - \gamma_G)}.
\]

Simplifying, we obtain \( \lim_{\theta \to \infty} r_M(\theta) = \frac{\Gamma(2 - \gamma_G)}{\Gamma(1 - \gamma_G)} = 1 - \gamma_G \). ■

Proposition 6 presents the asymptotic markup elasticity and the asymptotic expected markup for limit pricing (i.e. in the limit as \( \theta \to \infty \)). The asymptotic expected markup is the same as that found in Proposition 2 of Gabaix et al. (2016) and we recover their result that the markup elasticity converges to the tail index of \( G \), i.e. \( \varepsilon(\theta) \to \gamma_G \).

**Proposition 6.** Assume that Lemma 4 applies and \( G \) is well-behaved. In the limit as \( \theta \to \infty \), we have \( \varepsilon(\theta) \to \gamma_G \), where \( \gamma_G \) is the tail index of \( G \). The asymptotic expected markup is

\[
(50) \quad \mu(\theta) \sim_{\theta \to \infty} \frac{\Gamma(1 - \gamma_G)}{\theta g(G^{-1}(1 - 1/\theta))}.
\]

**Proof.** To derive the asymptotic markup elasticity, apply Lemma 5 and Proposition 4, using the fact that \( \lim_{\theta \to \infty} \varepsilon_D(\theta) = 1 \). To derive the asymptotic expected markup, starting
with (4) and rearranging yields

\begin{equation}
\mu(\theta) = \int_{x_0}^{x_1} \left( \frac{1-G(x)}{g(x)} \right) g(x) \theta e^{-\theta(1-G(x))} dx + \theta e^{-\theta} x_0 \quad (51)
\end{equation}

Letting \( \zeta(x) = \frac{1-G(x)}{g(x)} \) and applying Lemma 4, we obtain

\begin{equation}
\mu(\theta) \sim_{\theta \to \infty} \frac{\Gamma(\rho + 1)}{\theta g(G^{-1}(1 - 1/\theta))}. \quad (52)
\end{equation}

To determine \( \rho \), let \( \hat{\zeta}(t) = \frac{t}{g(G^{-1}(1-t))} \). Using Lemma A1 in the Appendix of Gabaix et al. (2016), we have \( \hat{\zeta}(t) \in RV_{\rho}^0 \) where \( \rho = 1 - (\gamma_G + 1) = -\gamma_G \). Therefore, we obtain (50).

As shown in Gabaix et al. (2016), all of the random utility models considered in that paper (e.g. Perloff and Salop, 1985; Sattinger, 1984; and Hart, 1985) feature equilibrium markups that are asymptotically proportional to the limit pricing markup, which is given by (50). For example, in the Perloff and Salop (1985) model, where firms set prices before observing the realizations of consumers' utility shocks, the equilibrium markup is asymptotically proportional to the limit pricing markup given by (50).

### 6 Examples

To bring our results to life, we present some examples for particular distributions. For each distribution, we consider two natural special cases regarding the value of the consumer’s outside option: \( z = 0 \) and \( z = x_0 \). In the first case, the consumer has no outside option. In the second case, the consumer’s outside option is equal to the minimum utility shock.

**Example 1.1. Exponential distribution with** \( z = x_0 \). Let \( G(x) = 1 - e^{-a(x-x_0)} \) for \( x \in [x_0, \infty) \) where \( a \in (0, \infty) \). Letting \( z = x_0 \), and using (4), we obtain

\begin{equation}
\mu(\theta) = \frac{1}{a}. \quad (53)
\end{equation}

Thus, the expected markup is constant and \( \varepsilon_\mu(\theta) = 0 \) (as is the case when \( n \) is fixed).
Example 1.2. Exponential distribution with $z = 0$. Let $G(x) = 1 - e^{-a(x-x_0)}$ for $x \in [x_0, \infty)$ where $a \in (0, \infty)$. Letting $x_0 = 1$ and $z = 0$, and using (4), we obtain

\begin{equation}
\mu(\theta) = \frac{1}{a} + \frac{\theta e^{-\theta}}{1 - e^{-\theta}}.
\end{equation}

Since $\frac{\theta e^{-\theta}}{1 - e^{-\theta}}$ is decreasing in $\theta$, the expected markup is always decreasing in the expected number of firms, i.e. $\mu'(\theta) < 0$. In the limit as $\theta \to \infty$, we have $\frac{\theta e^{-\theta}}{1 - e^{-\theta}} \to 0$ and therefore $\mu(\theta) \to \frac{1}{a}$ and $\varepsilon_{\mu}(\theta) \to 0$, the tail index of $G$.

Example 2.1. Pareto distribution with $z = x_0$. Let $G(x) = 1 - \left(\frac{x}{x_0}\right)^{-1/\lambda}$ for $x \in [x_0, \infty)$ where $\lambda \in (0, 1)$. Letting $z = x_0$, and using (4), we obtain

\begin{equation}
\mu(\theta) = \frac{\lambda x_0 \theta^\lambda \gamma(1 - \lambda, \theta)}{1 - e^{-\theta}}
\end{equation}

where $\gamma(s, z) \equiv \int_0^z t^{s-1}e^{-t} \, dt$, the Lower Incomplete Gamma Function. The expected markup is always increasing in the expected number of firms, $\theta$. In the limit as $\theta \to \infty$, we have $\varepsilon_{\mu}(\theta) \to \lambda > 0$, the tail index of the distribution of utility shocks $G$.

Example 2.2. Pareto distribution with $z = 0$. Let $G(x) = 1 - \left(\frac{x}{x_0}\right)^{-1/\lambda}$ for $x \in [x_0, \infty)$ where $\lambda \in (0, 1)$. Letting $x_0 = 1$ and $z = 0$, we obtain

\begin{equation}
\mu(\theta) = \frac{\lambda \theta^\lambda \gamma(1 - \lambda, \theta) + \theta e^{-\theta}}{1 - e^{-\theta}}.
\end{equation}

Letting $\varepsilon(s, z)$ denote the elasticity of $\gamma(s, z)$ with respect to $z$, it can be shown that

\begin{equation}
\varepsilon_{\mu}(\theta) = \frac{\lambda^2 + (1 + \lambda - \theta)\varepsilon(1 - \lambda, \theta)}{\lambda + \varepsilon(1 - \lambda, \theta)} - \frac{\theta e^{-\theta}}{1 - e^{-\theta}}.
\end{equation}

In this example, the expected markup $\mu(\theta)$ varies non-monotonically with the expected number of firms. In the limit as $\theta \to 0$, we have $\mu(\theta) \to E_{G}(x) - z$ and $\varepsilon_{\mu}(\theta) \to 0$. When the number of firms is relatively small, the markup elasticity is negative and the expected markup $\mu(\theta)$ is decreasing. As the number of firms increases, however, the markup elasticity eventually becomes positive and markups are eventually increasing in $\theta$. In the limit as
$\theta \to \infty$, we have $\varepsilon_\mu(\theta) \to \lambda > 0$, the tail index of $G$.

Figure I provides an illustration of the behavior of the markup elasticity for the case where $\lambda = 0.25$. In this example, competition is price-decreasing when the expected number of firms is less than around five. Eventually, however, when the expected number of firms is greater than around five, competition is price-increasing.

**Non-monotonicity of expected markup.** Importantly, whether or not markups are non-monotonic in the expected number of firms does not depend simply on properties of the distribution of the utility shocks, which influence how $E(M_n - S_n)$ varies with the realized number of firms. The behavior of the expected markup $\mu(\theta)$ also depends crucially on the *value of the consumer’s outside option*, $z$. The reason why consumers’ outside option is important is because there may be only one firm selling to the consumer in our environment. When there is only one firm, the expected markup is equal to $E_G(x) - z$, which clearly depends on the value of the consumer’s outside option $z$.

In Example 2.2, we assume that $z = 0$. In general, however, the expected markup is always a non-monotonic function of the expected number of firms whenever the distribution of utility shocks is Pareto and consumer’s outside option is strictly less than the minimum firm-specific utility shock, i.e. $z < x_0$. On the other hand, in the special case where $z = x_0$, i.e. when the consumer’s outside option is equal to the minimum utility shock (as in Example 2.1), there is no non-monotonicity at all. Instead, the expected markup $\mu(\theta)$ is monotonically increasing in the expected number of firms $\theta$.

## 7 Conclusion

This paper studies the effect of *expected competition* on markups in a random utility model where the number of competing firms is ex ante uncertain. There may be either no firms, one firm, or two or more firms competing for a single consumer. Prices are determined by “limit pricing”, i.e. the equilibrium markup equals the difference between the highest and second-highest utility shock. We derive simple, new expressions for the expected markup, the markup elasticity, and the consumer surplus in terms of the key object $M(\theta)$, the consumer’s expected utility as a function of the expected number of firms, $\theta$.

We show that our simple expression for the expected markup can be interpreted as
expressing the efficiency of firm entry under limit pricing. In turn, this simple expression reveals that the impact of competition on prices depends crucially on the curvature of the function $M(.)$. In particular, competition is price-increasing if and only if the coefficient of relative risk aversion of this function, $-M''(\theta)\theta/M'(\theta)$, is strictly less than the elasticity of demand with respect to $\theta$. Whether or not this is true depends not only on the properties of the distribution of utility shocks but also on the expected number of firms and the value of the consumer’s outside option. As a result, markups can vary non-monotonically with the expected number of firms. Importantly, however, we find that the consumer surplus is always strictly increasing in the expected number of firms due to consumers’ higher expected utility. This suggests that consumers are always better off when there are more competing firms, despite the fact that prices may sometimes be higher.

Our assumption that there is ex ante uncertainty regarding the exact number of firms competing for a particular consumer is more realistic in environments that feature various frictions (e.g. search frictions). Somewhat surprisingly, however, this approach does not add any greater complexity to the analysis. In fact, it yields significantly greater tractability and a remarkably simple expression for the expected markup, compared to an identical environment where the number of firms is fixed. This suggests that a similar approach may be fruitfully applied to many other problems in industrial organization in order to deliver greater tractability. We leave this as a potential avenue for future research.
References


Figure I: The markup elasticity $\varepsilon_\mu(\theta)$ as a function of the expected number of firms $\theta$ in Example 2.2 where $x_0 = 1$ and $z = 0$. The Pareto distribution of utility shocks has tail index $\lambda = 0.25$. 