The impact of competition on prices*

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Abstract

We study the effect of competition on markups in random utility models where prices are determined by "limit pricing", i.e. the symmetric equilibrium markup is equal to the difference between the highest and second-highest firm-specific utility shock. Gabaix, Laibson, Li, Li, Resnick, and de Vries (2016) show that greater competition (more firms) is *asymptotically* price-increasing when the distribution of utility shocks is fat tailed. We study the behavior of markups when the number of competing firms is *finite* but stochastic. The impact of competition on prices depends on the curvature of $M(\theta)$, the consumer's *expected utility* as a function of the expected number of firms, θ. In particular, competition is price-increasing if and only if the *coefficient of relative risk aversion* of this function, $-M''(\theta)\theta/M'(\theta)$, is strictly less than the elasticity of demand with respect to θ. Whether or not this condition holds depends not only on the distribution of utility shocks but also on the expected number of firms and the consumer's outside option. For example, when the distribution of utility shocks is Pareto, markups typically vary *non-monotonically* with the expected number of firms.

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1 Introduction

How do markups vary with the degree of competition between firms? This paper studies the impact of competition on prices in random utility models where consumer choice is determined by firm-specific utility shocks or "noise" shocks (Anderson, de Palma, and Thisse, 1992). We are interested in the question, when are markups increasing or decreasing in the number of competing firms? Gabaix, Laibson, Li, Li, Resnick, and de Vries (2016) consider a broad class of symmetric random utility models (e.g. Perloff and Salop, 1985; Sattinger, 1984; and Hart, 1985) and use extreme value theory to examine the impact of competition on prices when the number of firms becomes large. An important result is that, in the limit as the number of firms becomes infinite, the elasticity of markups with respect to the number of firms converges to the *tail index* γ of the distribution of firm-specific utility shocks (a measure of tail fatness). For example, if the distribution of utility shocks has positive tail index $\gamma > 0$, markups are asymptotically increasing in the number of firms.[1]

While the asymptotic results in Gabaix et al. (2016) are insightful and elegant, a key limitation is that they are only a useful approximation for markets that feature a large number of firms. For this reason, our paper considers the behavior of markups in an alternative environment where the number of firms is *finite* but stochastic. This approach achieves two things. First, it delivers greater realism. In many real-world environments, both consumers and firms may be ex ante uncertain about the exact number of competing firms. When calculating the ex ante *expected markup*, we must therefore take into consideration the uncertainty about both the random utility shocks and the number of competing firms. Second, it yields greater tractability compared to an environment with a fixed number of firms. The tractability of the framework allows us to provide simple, general results regarding the *expected markup* $\mu(\theta)$ as a function of the *expected number of firms*, θ.

We consider an environment where firms set prices using *limit pricing* (sometimes referred to as Bertrand competition).[2] When there are at least two firms, the symmetric equilibrium markup (i.e. price minus marginal cost, $p - c$) is equal to the difference between the highest and the second-highest firm-specific utility shock. When there is exactly one firm, the markup is equal to the difference between the firm-specific utility shock and the consumer's outside option (which may be zero). We focus on this type of pricing for three reasons. First, Gabaix et al. (2016) show that the equilibrium markups for all of the random utility models they

[1] If the distribution has negative tail index $\gamma < 0$, on the other hand, markups are asymptotically decreasing in the number of firms. If $\gamma = 0$, markups are relatively insensitive to changes in the degree of competition as the number of firms becomes large. See Gabaix et al. (2016) for details.

[2] Whether the term "limit pricing" is correct here is controversial. We bypass this debate and simply follow Gabaix et al. (2016) in using this phrase to describe this type of pricing.

consider are asymptotically proportional to the limit pricing markup, revealing a common "limit pricing" logic underlying this class of models. Second, the expected limit pricing markup is mathematically identical to the winner's expected surplus in a second-price auction and therefore all of the key results in this paper regarding markups can be applied directly to auctions. Third, we will see that this type of pricing delivers an *efficient* level of entry of firms by ensuring firms are paid their marginal contribution to the social surplus.

The impact of a greater number of firms on the expected markup depends on two effects that work in opposite directions: the *competition effect*; and the *value creation effect*. The competition effect reflects the *negative* effect of competition on markups through decreasing firms' share of the expected "value created" by firms, i.e. the expected value of the highest utility shock. The value creation effect captures the *positive* effect of competition on markups through increasing the value created by firms. In principle, either effect may dominate.

We derive a simple expression that relates the expected markup $\mu(\theta)$ to the consumer's *expected utility*, $M(\theta)$, and the *expected demand*, $D(\theta)$. Both of these are functions of the expected number of firms, θ, while the function $M(.)$ also depends on both the distribution of utility shocks and the consumer's outside option. We show that the expected markup is given by $\mu(\theta) = M'(\theta)/D(\theta)$. As a result, limit pricing markups induce an efficient level of entry of firms. Since the expected payoff of a firm is $\mu(\theta)D(\theta) = M'(\theta)$, firms are paid for their marginal contribution to the consumer's expected utility and thus for their marginal contribution to the social surplus. This connection between the expected markup and the consumer's expected utility delivers a simple, general condition under which competition is either *price-increasing* ($\mu'(\theta) > 0$) or *price-decreasing* ($\mu'(\theta) < 0$). Competition is price-increasing if and only if the *coefficient of relative risk aversion* of the consumer's expected utility function, $-M''(\theta)\theta/M'(\theta)$, is strictly less than $\varepsilon_D(\theta)$, the elasticity of the expected demand with respect to θ. Intuitively, as the number of firms rises, the marginal increase $M'(\theta)$ in the consumer's expected utility decreases since $M(.)$ is concave and $M''(\theta) < 0$, but if the rate of decrease in $M'(\theta)$ is sufficiently low, i.e. if $M(.)$ is not *too* concave relative to the demand elasticity, then competition is price-increasing and $\mu'(\theta) > 0$.

As discussed, this paper is most closely related to Gabaix et al. (2016). Our approach is complementary to that paper because *(i)* the number of firms is finite but stochastic, rather than infinite; and *(ii)* we focus on "limit pricing" markups. More broadly, this paper is related to the literature on oligopoly pricing, as presented in Vives (2001). More narrowly, this paper is related to a literature that considers the possibility of price-increasing competition. In a different framework, Chen and Riordan (2008) show that, under certain conditions, prices may be higher under duopoly than under monopoly and thus competition may be price-increasing. Weyl and Fabinger (2013) and Quint (2014) both focus on different questions, but point out that the comparative statics of pricing behavior depends crucially on log-concavity of the demand function. These results are indirectly related to the present paper because log-concavity of the demand function depends on properties of the distribution of utility shocks – in particular, whether the density of the distribution is log-concave (log-convex). In an earlier paper, Anderson, De Palma, and Nesterov (1995) consider the Perloff-Salop model and show that a sufficient condition for markups to be weakly decreasing in the number of firms is log-concavity of the density of the distribution of utility shocks.

Our results differ from the literature described above, for two main reasons. First, our criterion for markups to be increasing (decreasing) in the expected number of firms is local, not global (e.g. log-concavity). Second, the impact of competition on markups depends not only on properties of the distribution of utility shocks, but also on the expected number of firms and the value of the consumer's outside option. Intuitively, this is because it depends on the curvature of the function $M(.)$ that gives consumer's *expected utility*, which depends on both the expected number of firms and the consumer's outside option, as well as the distribution of shocks. For example, when the distribution of utility shocks is Pareto (which has a log-convex density), we find that competition is not always price-increasing. Instead, markups vary *non-monotonically* with the expected number of firms whenever the consumer's outside option is strictly less than the minimum utility shock. At first, when the expected number of firms is relatively low, competition is price-decreasing. Later, when the expected number of firms is sufficiently high, competition is price-increasing.

2 The model

Consider a single product market with a single consumer. The number of firms $n \in \mathbb{N}$ is stochastic. Specifically, the distribution of the number of firms is Poisson, i.e. the probability that there are n firms supplying the market is $P_n(\theta) = \frac{\theta^n e^{-\theta}}{n!}$ for all $n \in \mathbb{N}$. The expected number of firms supplying the market is θ, i.e. $E(n) = \theta$. We take θ to be *exogenous.*[3]

Firms are ex ante identical. Each firm can produce one unit of the good at marginal cost $c \geq 0$. After the number of competing firms n is realized, the consumer draws an i.i.d. valuation x_i for each firm $i \in \{1, 2, ..., n\}$ from an exogenous distribution with cdf $G(.)$.

Assumption 1. *The distribution of random utility shocks has a continuous, twice differentiable cdf $G(.)$ with pdf $g = G'$, support $[x_0, \overline{x}) \subseteq \mathbb{R}_+$, and a finite mean.*

[3] We take θ to be exogenous in order to focus our attention on the effect on markups of varying θ. However, it would be straightforward to endogenize the expected number of firms, θ. The Poisson distribution could also be endogenized, e.g. by considering mixed strategies of a large number of potential entrants.

To keep the environment as general as possible, we allow the possibility that the consumer can produce the good themselves at cost c and receive utility $z \in [0, x_0]$. The parameter z represents consumer's *outside option,* which may be zero.[4]

Firms set prices simultaneously, *after* observing the consumer's firm-specific utility shocks x_i for each $i \in \{1, 2, ..., n\}$. The consumer purchases one unit of the good, choosing the firm i that maximizes his net utility, $x_i - p_i$. Profits for the successful firm i are given by the markup, $\mu_i = p_i - c$. Without loss of generality, we normalize $c = 0$ throughout the paper.

Timing of events:

1. Number of firms n is realized.
2. Random utility shocks x_i are realized.
3. Firms observe shocks.
4. Firms set prices simultaneously.
5. Consumer makes purchase decision.
6. Production takes place.
7. Firm profits are realized.

3 Results

In this section, we present our main results regarding the behavior of equilibrium markups. First, we derive the equilibrium expected markup. Next, we derive a simple expression that relates the expected markup to the consumer's expected utility and the expected demand. Finally, we derive a simple, general condition under which markups are either increasing or decreasing in the expected number of competing firms.

3.1 Equilibrium markup

Let $\mathbf{x} = (x_1, ..., x_n)$, the realized values of the utility shocks. Consider the symmetric equilibrium where $p_i(\mathbf{x}) = p(\mathbf{x})$ for all firms $i \in \{1, 2, ..., n\}$. In equilibrium, when there are at least two firms, the consumer purchases from the firm with the highest valuation, $M_n = \max\{x_1, ..., x_n\}$, and the markup is equal to $M_n - S_n$, where S_n is the second-highest valuation. This is just enough to keep the second-best firm out of competition for the consumer. When there is exactly one firm, the consumer purchases from that firm and the markup is equal to $E_G(x) - z$. Following Gabaix et al. (2016), we call this *limit pricing.*

[4] In an auction setting, z represents the seller's own valuation of the good.

Given a fixed number $n \geq 2$ of firms, it is well known that the expected value of the difference between the first and second order statistic is

$$E(M_n - S_n) = E_{H_n}\left(\frac{1 - G(x)}{g(x)}\right), \tag{1}$$

where the expected value is taken with regard to the distribution of the first order statistic, $H_n(x) = (G(x))^n$. See, for example, McAfee and McMillan (1987). Therefore, when there are at least two firms, the expected markup is

$$\mu(n) = \int_{x_0}^{\overline{x}} nG(x)^{n-1}(1 - G(x))dx. \tag{2}$$

When there is exactly one firm, the expected markup is

$$\mu(1) = E_G(x) - z. \tag{3}$$

When there are no firms, we simply assume $\mu(0) = 0$.[5]

In our environment, where the number of competing firms n is stochastic and $n \sim P_n(\theta)$, we obtain the following expression for $\mu(\theta)$, the *expected markup* (conditional on $n \geq 1$).

Proposition 1. *If G satisfies Assumption 1, the equilibrium expected markup is*

$$\mu(\theta) = \frac{\int_{x_0}^{\overline{x}} \theta e^{-\theta(1-G(x))}(1 - G(x))dx + \theta e^{-\theta}(x_0 - z)}{1 - e^{-\theta}} \tag{4}$$

and the markup elasticity, $\varepsilon_\mu(\theta) \equiv \mu'(\theta)\theta/\mu(\theta)$, is

$$\varepsilon_\mu(\theta) = \frac{\int_{x_0}^{\overline{x}} \theta e^{-\theta(1-G(x))}(1 - \theta(1 - G(x)))(1 - G(x))dx + \theta e^{-\theta}(x_0 - z)(1 - \theta)}{\int_{x_0}^{\overline{x}} \theta e^{-\theta(1-G(x))}(1 - G(x))dx + \theta e^{-\theta}(x_0 - z)} - \frac{\theta e^{-\theta}}{1 - e^{-\theta}}. \tag{5}$$

Proof. We start with the following:

$$\mu(\theta) = \frac{\sum_{n=1}^{\infty} P_n(\theta)\mu(n)}{1 - e^{-\theta}}. \tag{6}$$

Substituting (2) and (3) into (6), and using the fact that $P_n(\theta) = \frac{\theta^n e^{-\theta}}{n!}$, we have

$$\mu(\theta) = \frac{\sum_{n=2}^{\infty} \frac{\theta^n e^{-\theta}}{n!} \int_{x_0}^{\overline{x}} nG(x)^{n-1}(1 - G(x))dx + \theta e^{-\theta}\left(E_G(x) - z\right)}{1 - e^{-\theta}}. \tag{7}$$

[5] This assumption is not important since our key object of interest is the expected markup conditional on $n \geq 1$, which does not depend on $\mu(0)$.

Reversing the integral and the summation on the left-hand side of the numerator in (7),

$$\mu(\theta) = \frac{\int_{x_0}^{\overline{x}} \sum_{n=2}^{\infty} \frac{\theta^n e^{-\theta}}{n!} nG(x)^{n-1}(1 - G(x))dx + \theta e^{-\theta}\left(E_G(x) - z\right)}{1 - e^{-\theta}}. \tag{8}$$

Rearranging and simplifying (8) yields

$$\mu(\theta) = \frac{\int_{x_0}^{\overline{x}} \theta e^{-\theta} \sum_{n=1}^{\infty} \frac{(\theta G(x))^n}{n!}(1 - G(x))dx + \theta e^{-\theta}\left(E_G(x) - z\right)}{1 - e^{-\theta}}, \tag{9}$$

and, using the fact that $\sum_{n=1}^{\infty} \frac{(\theta G(x))^n}{n!} = e^{\theta G(x)} - 1$, we obtain

$$\mu(\theta) = \frac{\int_{x_0}^{\overline{x}} \theta e^{-\theta}(e^{\theta G(x)} - 1)(1 - G(x))dx + \theta e^{-\theta}\left(E_G(x) - z\right)}{1 - e^{-\theta}}, \tag{10}$$

or, equivalently,

$$\mu(\theta) = \frac{\int_{x_0}^{\overline{x}} \theta e^{-\theta(1-G(x))}(1 - G(x))dx + \theta e^{-\theta}\left(E_G(x) - \int_{x_0}^{\overline{x}}(1 - G(x))dx - z\right)}{1 - e^{-\theta}}. \tag{11}$$

Given that G has a finite mean, we have $\lim_{x\to\overline{x}} x(1-G(x)) = 0$ and therefore $\int_{x_0}^{\overline{x}} xg(x)dx - \int_{x_0}^{\overline{x}}(1 - G(x))dx = x_0$ using integration by parts. Letting $E_G(x) - \int_{x_0}^{\overline{x}}(1 - G(x))dx = x_0$ in (11), we obtain (4). Finally, differentiating (4), we obtain

$$\begin{aligned}\mu'(\theta) \quad &= \quad \frac{\int_{x_0}^{\overline{x}} e^{-\theta(1-G(x))}(1 - \theta(1 - G(x)))(1 - G(x))dx + e^{-\theta}(x_0 - z)(1 - \theta)}{1 - e^{-\theta}} \\ &\quad - \frac{e^{-\theta}\left(\int_{x_0}^{\overline{x}} \theta e^{-\theta(1-G(x))}(1 - G(x))dx + \theta e^{-\theta}(x_0 - z)\right)}{(1 - e^{-\theta})^2}.\end{aligned} \tag{12}$$

Using the definition $\varepsilon_\mu(\theta) \equiv \mu'(\theta)\theta/\mu(\theta)$ and rearranging yields (5). ■

It is unclear whether the expected markup $\mu(\theta)$ is increasing or decreasing in the expected number of firms, or *degree of competition*. We are interested in the question, under what conditions do we have $\mu'(\theta) > 0$ or $\mu'(\theta) < 0$? By simply examining expressions (4) and (5), it is not easy to see when competition is either *price-increasing* or *price-decreasing*.

3.2 Consumer's expected utility

First, we derive expressions for the distribution of the consumer's utility and the expected value of the consumer's utility. When there is at least one firm, the consumer's utility equals the highest utility shock x among n draws when $n \sim P_n(\theta)$. When there are no firms, the consumer receives utility z, their outside option.

Let $H_n(.)$ be the cdf of the distribution of the maximum of n draws from $G(x)$, i.e. $H_n(x) \equiv (G(x))^n$. The distribution of the consumer's utility when $n \sim P_n(\theta)$ is denoted by $H(.;\theta)$ and it is given by the following:

$$H(x;\theta) = \begin{cases} \sum_{n=0}^{\infty} P_n(\theta)\,(G(x))^n & \text{if } x \in [z,\overline{x}) \\ 0 & \text{if } x \in [0,z) \end{cases} \tag{13}$$

Using $P_n(\theta) = \frac{\theta^n e^{-\theta}}{n!}$, it can be shown that[6]

$$H(x;\theta) = \begin{cases} e^{-\theta(1-G(x))} & \text{if } x \in [z,\overline{x}) \\ 0 & \text{if } x \in [0,z) \end{cases} \tag{14}$$

The distribution $H(.;\theta)$ has support $[x_0,\overline{x}) \cup \{z\}$. It features a mass point at z, since with probability $e^{-\theta}$ there are no firms and the consumer's utility equals their outside option, z.

Now let $M(\theta)$ denote the *expected utility* of the consumer, i.e. $M(\theta) \equiv E_H(x)$. Lemma 1 summarizes some key properties of the function $M(.)$.

Lemma 1. *If G satisfies Assumption 1, the consumer's expected utility is*

$$M(\theta) = \int_{x_0}^{\overline{x}} \theta e^{-\theta(1-G(x))} x g(x) dx + e^{-\theta} z \tag{15}$$

and the derivative of $M(.)$ is given by

$$M'(\theta) = \int_{x_0}^{\overline{x}} e^{-\theta(1-G(x))}(1-G(x))dx + e^{-\theta}(x_0 - z). \tag{16}$$

The function $M(.)$ has the following properties: (i) $M'(\theta) > 0$; (ii) $M''(\theta) < 0$; (iii) $M(0) = z$; (iv) $\lim_{\theta\to\infty} M(\theta) = \overline{x}$; (v) $\lim_{\theta\to\infty} M'(\theta) = 0$; and (vi) $\lim_{\theta\to 0} M'(\theta) = E_G(x) - z$.

Proof. Using $M(\theta) \equiv E_H(x)$ and expression (14) for the distribution $H(x;\theta)$, we obtain (15). *Part (i).* Applying Leibniz's integral rule to (15), we have

$$M'(\theta) = \int_{x_0}^{\overline{x}} x g(x) e^{-\theta(1-G(x))} dx - \int_{x_0}^{\overline{x}} \theta x g(x) e^{-\theta(1-G(x))}(1-G(x))dx. \tag{17}$$

[6] This uses the fact that $e^{-\theta} \sum_{n=0}^{\infty} \frac{(\theta G(x))^n}{n!} = e^{-\theta} e^{\theta G(x)} = e^{-\theta(1-G(x))}$.

By integration by parts on the right integral, and using the fact that $\lim_{x\to\overline{x}} x(1-G(x)) = 0$,

$$(18)\quad \int_{x_0}^{\overline{x}} \theta x g(x) e^{-\theta(1-G(x))}(1-G(x))dx = -x_0 e^{-\theta} - \int_{x_0}^{\overline{x}} e^{-\theta(1-G(x))}((1-G(x)) - xg(x))dx.$$

Substituting (18) into (17) yields (16), and clearly $M'(\theta) > 0$. *Part (ii).* Applying Leibniz' integral rule again, we obtain

$$(19)\qquad M''\theta) = -\left(\int_{x_0}^{\overline{x}} e^{-\theta(1-G(x))}(1-G(x))^2 dx + e^{-\theta}(x_0 - z)\right) < 0.$$

Parts (iii) and (v). It is clear that $M(0) = z$ and $\lim_{\theta\to\infty} M'(\theta) = 0$. *Part (iv).* Consider $\lim_{\theta\to\infty} M(\theta)$. Letting $t = 1 - G(x)$, we have $M(\theta) = \theta \int_0^1 e^{-\theta t} G^{-1}(1-t)dt + e^{-\theta} z$. Defining $G^{-1}(y) = 0$ for $y < 0$, we have $G^{-1}(1-t) = 0$ for $t > 1$ so $M(\theta) = \theta \int_0^\infty e^{-\theta t} G^{-1}(1-t)dt + e^{-\theta} z$. We can now apply the initial value theorem for Laplace transforms. This theorem states that for any piecewise continuous function $\phi(t)$, $\lim_{\theta\to\infty} \theta \int_0^\infty e^{-\theta t}\phi(t)dt = \lim_{t_0\to 0} \phi(t_0)$. So $\lim_{\theta\to\infty} M(\theta) = \lim_{t_0\to 0} G^{-1}(1-t_0) + 0 = G^{-1}(1) = \overline{x}$. *Part (vi).* Using (17), $\lim_{\theta\to 0} M'(\theta) = \lim_{\theta\to 0} \int_{x_0}^{\overline{x}} xg(x)e^{-\theta(1-G(x))}dx - z = \int_{x_0}^{\overline{x}} xg(x)dx - z = E_G(x) - z$. ■

3.3 Simple expression for expected markup

We now present a simple expression that relates the expected markup $\mu(\theta)$ to the consumer's expected utility $M(\theta)$ and the expected demand for a single firm's product. Let $D(\theta)$ denote the expected demand faced by single firm, i.e. the probability of a sale. Before we present Proposition 2, the following lemma gives us the expected demand $D(\theta)$.

Lemma 2. *If G satisfies Assumption 1, the expected demand is given by*

$$(20)\qquad D(\theta) = \frac{1 - e^{-\theta}}{\theta}.$$

Proof. For a fixed number of firms, it is well-known that the expected demand for a single firm's product in symmetric equilibrium is equal to

$$(21)\qquad D(n) = \int_{x_0}^{\overline{x}} g(x)(G(x))^{n-1}dx.$$

This is equal to the probability that the firm's utility shock is higher than that of all $n-1$ of the other firms. Since $D(n)n = \int_{x_0}^{\overline{x}} h_n(x)dx = 1$, we have $D(n) = 1/n$. Given that

$n \sim P_n(\theta)$, the expected demand is

$$D(\theta) = \sum_{n=1}^{\infty} P_n^f(\theta) \int_{x_0}^{\overline{x}} g(x)(G(x))^{n-1} dx \tag{22}$$

where $P_n^f(\theta) \equiv \frac{\theta^{n-1}e^{-\theta}}{(n-1)!}$, the probability that there is n firms from the perspective of *firms.* Reversing the order of the integral and the summation in (22), we have

$$D(\theta) = \int_{x_0}^{\overline{x}} g(x) \sum_{n=1}^{\infty} P_n^f(\theta)(G(x))^{n-1} dx. \tag{23}$$

Substituting $P_n^f(\theta) \equiv \frac{\theta^{n-1}e^{-\theta}}{(n-1)!}$ into (23) and rearranging,

$$D(\theta) = \int_{x_0}^{\overline{x}} g(x) e^{-\theta} \sum_{n=0}^{\infty} \frac{(\theta G(x))^n}{n!} dx. \tag{24}$$

Finally, using the fact that $\sum_{n=0}^{\infty} \frac{(\theta G(x))^n}{n!} = e^{\theta G(x)}$, we obtain

$$D(\theta) = \int_{x_0}^{\overline{x}} g(x) e^{-\theta(1-G(x))} dx. \tag{25}$$

Since $D(\theta)\theta = \int_{x_0}^{\overline{x}} \theta g(x) e^{-\theta(1-G(x))} dx = 1 - e^{-\theta}$, we have (20). ■

The next result presents a simple, general expression for the expected markup $\mu(\theta)$ as a function of the expected number of firms. Proposition 2 states that the expected markup $\mu(\theta)$ is equal to the marginal increase in the consumer's expected utility, $M'(\theta)$, divided by the expected demand, $D(\theta)$. Given that we have already derived the relevant expressions for $M'(\theta)$ and $D(\theta)$ in Lemmas 1 and 2, it is straightforward to prove this result.

Proposition 2. *If G satisfies Assumption 1, the expected markup is given by*

$$\mu(\theta) = \frac{M'(\theta)}{D(\theta)}, \tag{26}$$

where $M(\theta)$ is expected utility and $D(\theta)$ is expected demand.

Proof. Dividing expression (16) for $M'(\theta)$ from Lemma 1 by expression (20) for $D(\theta)$ from Lemma 2, we obtain expression (4) from Proposition 1.

To better understand the intuition behind the simple expression in Proposition 2, we can think about it in terms of the efficiency of firm entry, which we discuss below.

3.3.1 Efficient entry of firms

Suppose that the expected number of firms θ is not exogenous but is instead determined by a zero profit condition.[7] The expected payoff for a firm is $\Pi(\theta) \equiv D(\theta)\mu(\theta)$, i.e. the probability of a sale multiplied by the expected markup. If firms can enter at cost $k > 0$, the zero profit condition says $\Pi(\theta) = k$. Proposition 2 says that the expected payoff of a firm is equal to the marginal increase in the consumer's expected utility, i.e. $D(\theta)\mu(\theta) = M'(\theta)$. Therefore, $\Pi'(\theta) = M''(\theta) < 0$ and the zero profit condition implies there is a unique equilibrium expected number of firms θ^* given by $M'(\theta^*) = k$.[8]

Now suppose a social planner were to choose the expected number of firms θ in order to maximize the expected social surplus per consumer, $\Omega = M(\theta) - c - k\theta$. The planner's choice θ^P would also satisfy $M'(\theta^P) = k$. Therefore, $\theta^* = \theta^P$ and firm entry is *efficient* under limit pricing. Intuitively, firms are paid exactly for their marginal contribution to the consumer's expected utility, and thus for their marginal contribution to the social surplus. This is because the expected markup equals the difference between the highest and second-highest utility shock. In this way, the limit pricing markup $\mu(\theta)$ is precisely the *efficient markup* from the perspective of inducing the socially optimal level of firm entry.

3.4 Value creation and competition effects

We now introduce the notions of "value created" by firms and the "value share" of firms. First, let $v(\theta)$ denote the expected value of the highest firm-specific utility shock when there is at least one firm. We have

$$v(\theta) = \frac{\int_{x_0}^{\overline{x}} \theta e^{-\theta(1-G(x))} x g(x) dx}{1 - e^{-\theta}}. \tag{27}$$

We can interpret $v(\theta)$ as the expected *value created* by firms, which is increasing in the expected number of firms, θ. A higher number of firms implies a higher expected value of the highest firm-specific utility shock.

[7] For simplicity, we are considering a single consumer and the expected number of firms θ. However, we could consider an environment with a large number of consumers U and then determine the equilibrium number of entering firms V, which then determines the expected number of firms per consumer $\theta \equiv V/U$.

[8] Anderson et al. (1995) consider the Perloff-Salop model and show that log-concavity is a sufficient condition for the existence of an equilibrium when there is free entry of firms. In our environment, this assumption is not required for determining firm entry since expected profits are always decreasing in the number of firms, i.e. $\Pi'(\theta) < 0$, even in cases where the expected markup is increasing in θ.

Now define $\alpha(\theta) \equiv \mu(\theta)/v(\theta)$, the *value share* of firms. Using (4) and (27), we obtain

$$\alpha(\theta) = \frac{\int_{x_0}^{\overline{x}} \theta e^{-\theta(1-G(x))}(1-G(x))dx + \theta e^{-\theta}(x_0 - z)}{\int_{x_0}^{\overline{x}} \theta e^{-\theta(1-G(x))} x g(x) dx}. \tag{28}$$

Before we present the next result, we provide some preliminary definitions.

Definition 1. *For $x \in [x_0, \overline{x})$, the* generalized hazard rate $\varepsilon_G(x)$ *is defined by*

$$\varepsilon_G(x) \equiv \frac{x g(x)}{1-G(x)}. \tag{29}$$

Definition 2. *We say that G is* IGHR *if and only if $\varepsilon'_G(x) \geq 0$ for all $x \in [x_0, \overline{x})$.*

Proposition 3 states that if G is IGHR, the value share of firms is always weakly *decreasing* in the expected number of firms. Intuitively, greater competition between firms implies that the successful firm always receives a lower *share* of the expected "value created".[9]

Proposition 3. *If G satisfies Assumption 1 and G is IGHR, the value share of firms $\alpha(\theta)$ is weakly decreasing in the expected number of firms θ, i.e. $\alpha'(\theta) \leq 0$.*

Proof. Letting $\bar{G}(x) = 1 - G(x)$, and differentiating (28), $\frac{d}{d\theta}\alpha(\theta) \geq 0$ if and only if

$$\begin{aligned} &\int_{x_0}^{\overline{x}} e^{-\theta \bar{G}(x)} \bar{G}(x) x g(x) dx \int_{x_0}^{\overline{x}} e^{-\theta \bar{G}(x)} \bar{G}(x) dx + e^{-\theta}(x_0 - z) \int_{x_0}^{\overline{x}} e^{-\theta \bar{G}(x)} \bar{G}(x) x g(x) dx \\ \leq &\int_{x_0}^{\overline{x}} e^{-\theta \bar{G}(x)} \bar{G}(x)^2 dx \int_{x_0}^{\overline{x}} e^{-\theta \bar{G}(x)} x g(x) dx + e^{-\theta}(x_0 - z) \int_{x_0}^{\overline{x}} e^{-\theta \bar{G}(x)} x g(x) dx. \end{aligned} \tag{30}$$

Since $\bar{G}(x) < 1$ for all $x > x_0$, it suffices to show that

$$\frac{\int_{x_0}^{\overline{x}} e^{-\theta \bar{G}(x)} \bar{G}(x) dx \int_{x_0}^{\overline{x}} e^{-\theta \bar{G}(x)} x g(x) \bar{G}(x) dx}{\int_{x_0}^{\overline{x}} e^{-\theta \bar{G}(x)} x g(x) dx \int_{x_0}^{\overline{x}} e^{-\theta \bar{G}(x)} \bar{G}(x)^2 dx} \leq 1. \tag{31}$$

Rearranging, and defining $\varepsilon_G^{-1}(x) \equiv 1/\varepsilon_G(x)$, (31) is equivalent to

$$\frac{\int_{x_0}^{\overline{x}} \varepsilon_G^{-1}(x) e^{-\theta \bar{G}(x)} x g(x) dx}{\int_{x_0}^{\overline{x}} e^{-\theta \bar{G}(x)} x g(x) dx} \leq \frac{\int_{x_0}^{\overline{x}} \varepsilon_G^{-1}(x) e^{-\theta \bar{G}(x)} x g(x) \bar{G}(x) dx}{\int_{x_0}^{\overline{x}} e^{-\theta \bar{G}(x)} x g(x) \bar{G}(x) dx}. \tag{32}$$

We can now apply Lemma 1 in the Appendix of Mangin (2017). This lemma states that, if $\alpha(.)$, $\beta(.)$ and $\varphi(.)$ are positive functions defined on $[x_0, \infty)$, and $\alpha'(x) \leq 0$ and $\beta'(x) <$

[9] The increasing generalized hazard rate (IGHR) condition is a very mild condition that is strictly weaker than the increasing hazard rate (IHR) condition. It is satisfied by almost all standard distributions.

0, then $\int_{x_0}^{\infty} \alpha(x)h(x)dx \leq \int_{x_0}^{\infty} \alpha(x)\hat{h}(x)dx$, where $h(x) \equiv \frac{\varphi(x)}{\int_{x_0}^{\infty} \varphi(x)dx}$ and $\hat{h}(x) \equiv \frac{\varphi(x)\beta(x)}{\int_{x_0}^{\infty} \varphi(x)\beta(x)dx}$. It is straightforward to verify that the proof generalizes to the case where $\overline{x}$ may be finite. Letting $\alpha(x) = \varepsilon_G^{-1}(x)$, $\varphi(x) = e^{-\theta\bar{G}(x)}xg(x)$, and $\beta(x) = \bar{G}(x)$, we have $\alpha(x) \geq 0$, $\varphi(x) \geq 0$ and $\beta(x) \geq 0$. Since G is IGHR, $\varepsilon_G'(x) \geq 0$ and thus $\alpha'(x) \leq 0$, and $\beta'(x) = -g(x) < 0$. Applying this lemma yields (32) and thus inequality (31) is proven and $\frac{d}{d\theta}\alpha(\theta) \geq 0$. ■

We can express the expected markup as the following product of two components:

$$\mu(\theta) = \alpha(\theta)\upsilon(\theta). \tag{33}$$

There are two channels through which an increase in the expected number of firms affects the expected markup. The first is the *competition effect* and the second is the *value creation effect*. Using (33), we can express the markup elasticity as a sum of two components:

$$\varepsilon_\mu(\theta) = \underbrace{\frac{\alpha'(\theta)\theta}{\alpha(\theta)}}_{\text{competition effect }(<0)} + \underbrace{\frac{\upsilon'(\theta)\theta}{\upsilon(\theta)}}_{\text{value creation effect }(>0)}. \tag{34}$$

Competition effect. The first component captures the *negative* effect of competition on markups. Greater competition has a negative effect on markups by decreasing firms' value share, $\alpha(\theta)$, which is weakly *decreasing* in the expected number of firms, i.e. $\alpha'(\theta) \leq 0$.

Value creation effect. The second component captures the *positive* effect of competition on markups. Greater competition has a positive effect on markups through increasing the expected "value created" by firms, $\upsilon(\theta)$, which is strictly *increasing* in the expected number of firms, i.e. $\upsilon'(\theta) > 0$.

Since these two effects of greater competition – a lower value share $\alpha(\theta)$ and greater value created $\upsilon(\theta)$ – work in opposite directions, competition may be either *price-increasing* ($\mu'(\theta) > 0$) or *price-decreasing* ($\mu'(\theta) < 0$).

3.5 When is competition price-increasing?

Gabaix et al. (2016) show that, in the limit as the number of firms $n \to \infty$, the markup elasticity $\varepsilon_\mu(n)$ converges to the *tail index* γ_G of the distribution of random utility shocks G. Competition is therefore either *asymptotically price-increasing* (i.e. $\mu'(n) > 0$ as $n \to \infty$) or *asymptotically price-decreasing* (i.e. $\mu'(n) < 0$ as $n \to \infty$) depending on whether the tail index is greater than or less than zero, i.e. whether the distribution is fat-tailed or not.

We now present some general results regarding whether competition is price-increasing or price-decreasing. These results apply for any *finite* expected number of firms, and we recover the asymptotic results of Gabaix et al. (2016) as a limiting case. As we will see, whether competition is price-increasing or price-decreasing depends not only on the characteristics of the underlying distribution of random utility shocks G, but also on the expected number of firms, θ, and the value of the consumer's outside option, z.

Proposition 4 presents a simple expression for the markup elasticity, $\varepsilon_\mu(\theta)$, which yields a general condition under which competition is price-increasing. Proposition 4 features a measure of the *local* curvature, or degree of concavity, of the function $M(.)$, which gives the expected utility of the consumer when the expected number of firms is θ . It is essentially the Arrow-Pratt *coefficient of relative risk aversion* of the function $M(.)$ at θ.[10] Proposition 4 also features the *demand elasticity,* $\varepsilon_D(\theta)$, i.e. the elasticity of the "demand" function $D(.)$ with respect to the expected number of firms, $\varepsilon_D(\theta) \equiv -D'(\theta)\theta/D(\theta)$.

Proposition 4. *If G satisfies Assumption 1, the markup elasticity is*

$$\varepsilon_\mu(\theta) = \varepsilon_D(\theta) - r_M(\theta), \tag{35}$$

where $\varepsilon_D(\theta)$ is the demand elasticity and

$$r_M(\theta) \equiv \frac{-M''(\theta)\theta}{M'(\theta)}. \tag{36}$$

The expected markup $\mu(\theta)$ is strictly increasing in the expected number of firms, i.e. $\mu'(\theta) > 0$, if and only if $r_M(\theta) < \varepsilon_D(\theta)$.

Proof. First, we derive the elasticity of the unconditional expected markup, $\mu_0(\theta)$, across *all* possible outcomes $n \geq 0$. Since $\mu(\theta) = \mu_0(\theta)/(1 - e^{-\theta})$, Proposition 1 implies that

$$\mu_0(\theta) = \int_{x_0}^{\overline{x}} \theta e^{-\theta(1-G(x))}(1 - G(x))dx + \theta e^{-\theta}(x_0 - z). \tag{37}$$

Using expression (16) for $M'(\theta)$ from Lemma 1, we obtain $\mu_0(\theta) = M'(\theta)\theta$. Differentiating,

$$\mu_0'(\theta) = M''(\theta)\theta + M'(\theta). \tag{38}$$

Rearranging (38) using definition (36), we have

$$\mu_0'(\theta) = (1 - r_M(\theta))\, M'(\theta). \tag{39}$$

[10] Of course, $M(.)$ is a function of the expected number of firms, not a standard utility function here.

Next, substituting $M'(\theta) = \mu_0(\theta)/\theta$ into (39), we obtain

$$\frac{\mu_0'(\theta)\theta}{\mu_0(\theta)} = 1 - r_M(\theta). \tag{40}$$

Since $\mu(\theta) = \mu_0(\theta)/(1 - e^{-\theta})$, we can write

$$\varepsilon_\mu(\theta) = \frac{\mu_0'(\theta)\theta}{\mu_0(\theta)} - \frac{\theta e^{-\theta}}{1 - e^{-\theta}} \tag{41}$$

and therefore, using (40), we have

$$\varepsilon_\mu(\theta) = 1 - r_M(\theta) - \frac{\theta e^{-\theta}}{1 - e^{-\theta}}. \tag{42}$$

Finally, it is straightforward to show that

$$\varepsilon_D(\theta) = 1 - \frac{\theta e^{-\theta}}{1 - e^{-\theta}} \tag{43}$$

and thus we obtain (35). Therefore, we have $\mu'(\theta) > 0$ if and only if $r_M(\theta) < \varepsilon_D(\theta)$. ■

The intuition behind Proposition 4 is the following. As the number of firms rises, the expected value of the highest valuation, $M(\theta)$, increases. However, the marginal increase $M'(\theta)$ in the expected value $M(\theta)$ is decreasing in θ since $M''(\theta) < 0$. If the rate of decrease in $M'(\theta)$ is sufficiently low, i.e. if $M''(\theta)$ is not *too* negative and $r_M(\theta)$ is not too high relative to the demand elasticity $\varepsilon_D(\theta)$ (i.e. $M(.)$ is not *too* concave), then $\mu'(\theta) > 0$. Intuitively, this is because the *value creation* effect, i.e. the positive effect of more firms on the expected value created, $M(\theta)$, may be sufficiently high that it dominates the *competition* effect, i.e. the negative effect of more firms on the firms' *share* of the value created, $\alpha(\theta)$.

3.6 Asymptotic markup

We now show that our results are consistent with those of Gabaix et al. (2016) by showing that their asymptotic results can be recovered from our expressions by taking the limit as $\theta \to \infty$. We first derive Lemma 3, which is a variant of Theorem 3 of Gabaix et al. (2016) that is adapted to the present environment.

Before presenting this general lemma, we define the notion of *regular variation*.

Definition 3. *We say that a function $h : \mathbb{R}_+ \to \mathbb{R}$ is* regularly varying *at zero with index ρ, and denote this by $h \in RV_\rho^0$, if and only if h is strictly positive in a neighborhood of zero* *and, for all $\lambda > 0$, we have*

$$\lim_{t \to 0} \frac{h(\lambda t)}{h(t)} = \lambda^{\rho}. \tag{44}$$

Aside from notational differences, Lemma 3 is identical to Theorem 3 of Gabaix et al. (2016), except that here $H(.;\theta)$ is the distribution of the consumer's expected utility when n is stochastic (not fixed), i.e. $n \sim P_n(\theta)$, and the consumer's outside option is z. We also restrict attention to the case where $\zeta(x) \geq 0$.

Lemma 3. *Let G be a differentiable cdf with support $(x_0, \overline{x})$ that is strictly increasing in a left neighborhood of $\overline{x}$. Let $\zeta : (x_0, \overline{x}) \cup \{0\} \to \mathbb{R}^+$ be a function that satisfies $\zeta(x) \geq 0$ for all $x \in (x_0, \overline{x})$. Suppose that $\hat{\zeta}(t) \equiv \zeta(G^{-1}(1-t)) \in RV_{\rho}^{0}$ with $\rho > -1$, and that $\int_{x_0}^{\overline{x}} |\zeta(x)g(x)|\, dx < \infty$. Then, as $\theta \to \infty$, we have*

$$E_H(\zeta(x)) = \int_{x_0}^{\overline{x}} \zeta(x)g(x)\theta e^{-\theta(1-G(x))}dx + e^{-\theta}\zeta(z) \sim \zeta\left(G^{-1}\left(1 - \frac{1}{\theta}\right)\right)\Gamma(\rho+1) \tag{45}$$

where $\Gamma(t) \equiv \int_0^{\infty} y^{t-1}e^{-y}dy$ is the Gamma function.

Proof. Consider the integral $E_H(\zeta(x)) = \int_{x_0}^{\overline{x}} \zeta(x)g(x)\theta e^{-\theta(1-G(x))}dx$ where we assume that $\zeta(x) \geq 0$ and $\hat{\zeta}(t) \equiv \zeta(G^{-1}(1-t)) \in RV_{\rho}^{0}$ with $\rho > -1$, and $\int_{x_0}^{\overline{x}} |\zeta(x)g(x)|\, dx < \infty$. Changing variables by letting $t = 1 - G(x)$ and rewriting yields

$$E_H(\zeta(x)) = \int_0^1 \theta e^{-\theta t}\zeta(G^{-1}(1-t))dt + e^{-\theta}\zeta(z). \tag{46}$$

Rewriting, this is equivalent to

$$E_H(\zeta(x)) = \int_0^1 \theta e^{-(\theta-1)t}\zeta(G^{-1}(1-t))e^{-t}dt + e^{-\theta}\zeta(z). \tag{47}$$

Now define $h(t) \equiv \zeta(G^{-1}(1-t))e^{-t}$ and $H(t) \equiv \int_0^t h(y)dy$. Letting $\theta - 1 = \theta'$, we have

$$E_H(\zeta(x)) = \int_0^1 \theta e^{-\theta' t}h(t)dt = \int_0^1 \theta e^{-\theta' t}dH(t) + e^{-\theta}\zeta(z). \tag{48}$$

Defining $\hat{h}(t) = h(t)$ for all $t \in [0,1]$ and $\hat{h}(t) = 0$ for all $t \in (1, \infty)$, and $\hat{H}(t) \equiv \int_0^t \hat{h}(y)dy$,

$$E_H(\zeta(x)) = \int_0^{\infty} \theta e^{-\theta' t}d\hat{H}(t) + e^{-\theta}\zeta(z). \tag{49}$$

We can apply Karamata's Tauberian Theorem since $\hat{H}(t)$ is weakly positive and weakly increasing in t. This theorem says that if $\hat{H}(t) \in RV_\alpha^0$ then as $\theta' \to \infty$ we have

$$\int_0^\infty e^{-\theta' t} d\hat{H}(t) \sim \hat{H}(1/\theta')\Gamma(\alpha+1). \tag{50}$$

Now, since $\hat{\zeta}(t) \equiv \zeta(G^{-1}(1-t)) \in RV_\rho^0$ with $\rho > -1$ by assumption, we have $h(t) \equiv \zeta(G^{-1}(1-t))e^{-t} \in RV_\rho^0$ with $\rho > -1$ since $e^{-t} \in RV_0^0$, and therefore also $\hat{h}(t) \in RV_\rho^0$ with $\rho > -1$. By Lemma A1.6 of Gabaix et al (2016), this implies that $\hat{H}(t) \equiv \int_0^t \hat{h}(y)dy \in RV_{\rho+1}^0$ and therefore $\alpha = \rho + 1$, so we have

$$\int_0^\infty e^{-\theta' t} d\hat{H}(t) \sim \hat{H}(1/\theta')\Gamma(\rho+2) \tag{51}$$

as $\theta' \to \infty$. By Lemma A1.6 of Gabaix et al (2016), we also have $\lim_{x\to 0} \frac{x\hat{h}(x)}{\hat{H}(x)} = \rho + 1$, and thus $\hat{H}(x) \sim x\hat{h}(x)/(\rho+1)$ as $x \to 0$. Therefore as $\theta' \to \infty$ we have

$$\int_0^\infty e^{-\theta' t} d\hat{H}(t) \sim \frac{1}{\theta'} \frac{\hat{h}(1/\theta')\Gamma(\rho+2)}{\rho+1}. \tag{52}$$

Since $\Gamma(\rho+2)/(\rho+1) = \Gamma(\rho+1)$ and $\hat{h}(t) = \zeta(G^{-1}(1-t))e^{-t}$ for all $t \in [0,1]$,

$$\int_0^\infty e^{-\theta' t} d\hat{H}(t) \sim \frac{1}{\theta'}\zeta\left(G^{-1}\left(1 - \frac{1}{\theta'}\right)\right) e^{-1/\theta'}\Gamma(\rho+1). \tag{53}$$

Finally, using the fact that $\theta' = \theta - 1$, in the limit as $\theta \to \infty$ we have

$$E_H(\zeta(x)) = \int_0^\infty \theta e^{-\theta' t} d\hat{H}(t) + e^{-\theta}\zeta(z) \sim \zeta\left(G^{-1}\left(1 - \frac{1}{\theta}\right)\right)\Gamma(\rho+1). \ \blacksquare \tag{54}$$

Before we present the next lemma, we provide a preliminary definition.

Definition 4. *We say that G is* well-behaved *if and only if it satisfies Assumption 1 and*

$$\lim_{x\to\overline{x}} \frac{1-G(x)}{g(x)} = a \qquad \textit{where } a \in \mathbb{R}^+ \cup \{+\infty\} \tag{55}$$

and G has finite tail index γ_G *given by*

$$\lim_{x\to\overline{x}} \frac{d}{dx}\left(\frac{1-G(x)}{g(x)}\right) = \gamma_G \quad \textit{where } \gamma_G \in \mathbb{R}. \tag{56}$$

We can now use Lemma 3 to derive the following result regarding the asymptotic behavior of the measure of local curvature, $r_M(\theta)$, of the expected utility function, $M(.)$. Lemma 4 states that, in the limit as the expected number of firms becomes large, we have $r_M(\theta) \to 1-\gamma_G$. This result is used here to prove Proposition 5, but it is interesting in its own right because it says that the *tail index* of the distribution of utility shocks – which is a measure of tail fatness – is equal to one minus the asymptotic value of $r_M(\theta)$ – a measure of local curvature of the consumer's expected utility function, $M(.)$.

Lemma 4. *Assume that Lemma 3 applies and G is well-behaved. In the limit as $\theta \to \infty$, we have $r_M(\theta) \to 1-\gamma_G$, where γ_G is the tail index of G.*

Proof. Starting with definition (36) of $r_M(\theta)$, and using (16) and (19),

$$\lim_{\theta\to\infty} r_M(\theta) = \lim_{\theta\to\infty} \frac{\int_{x_0}^{\overline{x}} \theta e^{-\theta(1-G(x))}(1-G(x))^2 dx + \theta e^{-\theta}(x_0 - z)}{\int_{x_0}^{\overline{x}} e^{-\theta(1-G(x))}(1-G(x)) dx + e^{-\theta}(x_0 - z)}. \tag{57}$$

Rearranging (57) and simplifying, this is equivalent to

$$\lim_{\theta\to\infty} r_M(\theta) = \lim_{\theta\to\infty} \frac{\theta \int_{x_0}^{\overline{x}} \left(\frac{(1-G(x))^2}{g(x)}\right) g(x)\theta e^{-\theta(1-G(x))} dx}{\int_{x_0}^{\overline{x}} \left(\frac{1-G(x)}{g(x)}\right) g(x)\theta e^{-\theta(1-G(x))} dx}. \tag{58}$$

Applying Lemma 3 to the numerator of (58), where $\zeta(x) = \frac{(1-G(x))^2}{g(x)}$, we have

$$\theta \int_{x_0}^{\overline{x}} \frac{(1-G(x))^2}{g(x)} g(x)\theta e^{-\theta(1-G(x))} dx \sim_{\theta\to\infty} \frac{\theta\left(1-G\left(G^{-1}\left(1-1/\theta\right)\right)\right)^2}{g(G^{-1}\left(1-1/\theta\right))}\Gamma(\rho_1+1). \tag{59}$$

To determine ρ_1, let $\hat{\zeta}(t) \equiv \frac{t^2}{g(G^{-1}(1-t))}$. Using Lemma A1 in the Appendix of Gabaix et al. (2016), we have $\hat{\zeta}(t) \in RV_\rho^0$ where $\rho_1 = 2-(\gamma_G+1) = 1-\gamma_G$. Similarly, applying Lemma 3 to the denominator of (58), where $\zeta(x) = \frac{1-G(x)}{g(x)}$, we obtain

$$\int_{x_0}^{\overline{x}} \frac{1-G(x)}{g(x)} g(x)\theta e^{-\theta(1-G(x))} dx \sim_{\theta\to\infty} \frac{\left(1-G\left(G^{-1}\left(1-1/\theta\right)\right)\right)}{g(G^{-1}\left(1-1/\theta\right))}\Gamma(\rho_2+1). \tag{60}$$

To determine ρ_2, let $\hat{\zeta}(t) = \frac{t}{g(G^{-1}(1-t))}$. Using Lemma A1 in the Appendix of Gabaix et al. (2016), we have $\hat{\zeta}(t) \in RV_\rho^0$ where $\rho_2 = 1-(\gamma_G+1) = -\gamma_G$. Therefore,

$$\lim_{\theta\to\infty} r_M(\theta) = \lim_{\theta\to\infty} \theta\left(1-G\left(G^{-1}\left(1-1/\theta\right)\right)\right)\frac{\Gamma(2-\gamma_G)}{\Gamma(1-\gamma_G)}. \tag{61}$$

Simplifying, we obtain

$$\lim_{\theta\to\infty} r_M(\theta) = \frac{\Gamma(2-\gamma_G)}{\Gamma(1-\gamma_G)} = 1-\gamma_G. \blacksquare \tag{62}$$

Proposition 5 presents the *asymptotic markup elasticity* and the *asymptotic expected markup* for limit pricing (i.e. in the limit as $\theta \to \infty$). The asymptotic expected markup is the same as that found in Proposition 2 of Gabaix et al. (2016) and we recover their result that the markup elasticity converges to the *tail index* of G, i.e. $\varepsilon_\mu(\theta) \to \gamma_G$.

Proposition 5. *Assume that Lemma 3 applies and G is well-behaved. In the limit as $\theta \to \infty$, we have $\varepsilon_\mu(\theta) \to \gamma_G$, where γ_G is the tail index of G. The asymptotic expected markup is*

$$\mu(\theta) \sim_{\theta\to\infty} \frac{\Gamma(1-\gamma_G)}{\theta g\left(G^{-1}(1-1/\theta)\right)}, \tag{63}$$

where $\Gamma(t) \equiv \int_0^\infty y^{t-1}e^{-y}dy$ is the Gamma function.

Proof. To derive the asymptotic markup elasticity, apply Lemma 4 and Proposition 4, using the fact that $\lim_{\theta\to\infty} \varepsilon_D(\theta) = 1$. To derive the asymptotic expected markup, starting with (37) and rearranging, we have

$$\mu(\theta) = \frac{\int_{x_0}^{\overline{x}} \left(\frac{1-G(x)}{g(x)}\right) g(x)\theta e^{-\theta(1-G(x))}dx + \theta e^{-\theta}x_0}{1-e^{-\theta}}. \tag{64}$$

Letting $\zeta(x) = \frac{1-G(x)}{g(x)}$ and applying Lemma 3, we obtain

$$\mu(\theta) \sim_{\theta\to\infty} \frac{\Gamma(\rho+1)}{\theta g(G^{-1}(1-1/\theta))}. \tag{65}$$

To determine ρ, let $\hat{\zeta}(t) = \frac{t}{g(G^{-1}(1-t))}$. Using Lemma A1 in the Appendix of Gabaix et al. (2016), we have $\hat{\zeta}(t) \in RV_\rho^0$ where $\rho = 1-(\gamma_G+1) = -\gamma_G$. Therefore, we obtain (63). ■

As shown in Gabaix et al. (2016), all of the random utility models considered in that paper (e.g. Perloff and Salop, 1985; Sattinger, 1984; and Hart, 1985) feature equilibrium markups that are asymptotically proportional to the limit pricing markup. As the authors point out, expression (63) also represents the expected surplus for the winner of a second-price auction in the limit as the number of bidders becomes large.

4 Examples

To bring these results to life, we present some examples for particular distributions. For each distribution, we consider two natural special cases regarding the value of the consumer's outside option: $z = 0$ and $z = x_0$. In the first case, the consumer has no outside option. In the second case, the consumer's outside option is equal to the minimum utility shock.

Example 1.1. Exponential distribution with $z = x_0$. Let $G(x) = 1 - e^{-a(x-x_0)}$ for $x \in [x_0, \infty)$ where $a \in (0, \infty)$. Letting $z = x_0$, and using (4), we obtain

$$\mu(\theta) = \frac{1}{a}. \tag{66}$$

Thus, the expected markup is *constant* and $\varepsilon_\mu(\theta) = 0$ (as is the case when n is fixed).

Example 1.2. Exponential distribution with $z = 0$. Let $G(x) = 1 - e^{-a(x-x_0)}$ for $x \in [x_0, \infty)$ where $a \in (0, \infty)$. Letting $x_0 = 1$ and $z = 0$, and using (4), we obtain

$$\mu(\theta) = \frac{1}{a} + \frac{\theta e^{-\theta}}{1 - e^{-\theta}}. \tag{67}$$

Since $\frac{\theta e^{-\theta}}{1-e^{-\theta}}$ is decreasing in θ, the expected markup is always *decreasing* in the expected number of firms, i.e. $\mu'(\theta) < 0$. In the limit as $\theta \to \infty$, we have $\frac{\theta e^{-\theta}}{1-e^{-\theta}} \to 0$ and therefore $\mu(\theta) \to \frac{1}{a}$ and $\varepsilon_\mu(\theta) \to 0$, the tail index of G.

Example 2.1. Pareto distribution with $z = x_0$. Let $G(x) = 1 - \left(\frac{x}{x_0}\right)^{-1/\lambda}$ for $x \in [x_0, \infty)$ where $\lambda \in (0, 1)$. Letting $z = x_0$, and using (4), we obtain

$$\mu(\theta) = \frac{\lambda x_0 \theta^\lambda \gamma(1-\lambda, \theta)}{1 - e^{-\theta}} \tag{68}$$

where $\gamma(s, z) \equiv \int_0^z t^{s-1} e^{-t}\, dt$, the Lower Incomplete Gamma Function. The expected markup is always *increasing* in the expected number of firms, θ. In fact, the expected markup is a constant share of the expected value created, $v(\theta)$. That is, $\mu(\theta) = \lambda v(\theta)$, or $\alpha(\theta) = \lambda$. In the limit as $\theta \to \infty$, we have $\varepsilon_\mu(\theta) \to \lambda$, the tail index of G, where $\lambda > 0$.

Example 2.2. Pareto distribution with $z = 0$. Let $G(x) = 1 - \left(\frac{x}{x_0}\right)^{-1/\lambda}$ for $x \in [x_0, \infty)$ where $\lambda \in (0, 1)$. Letting $x_0 = 1$ and $z = 0$, we obtain

$$\mu(\theta) = \frac{\lambda \theta^\lambda \gamma(1-\lambda, \theta) + \theta e^{-\theta}}{1 - e^{-\theta}}. \tag{69}$$

Letting $\varepsilon(s, z)$ denote the elasticity of $\gamma(s, z)$ with respect to z, it can be shown that

$$\varepsilon_\mu(\theta) = \frac{\lambda^2 + (1 + \lambda - \theta)\varepsilon(1-\lambda, \theta)}{\lambda + \varepsilon(1-\lambda, \theta)} - \frac{\theta e^{-\theta}}{1 - e^{-\theta}}. \tag{70}$$

In this example, the expected markup $\mu(\theta)$ varies *non-monotonically* with the expected

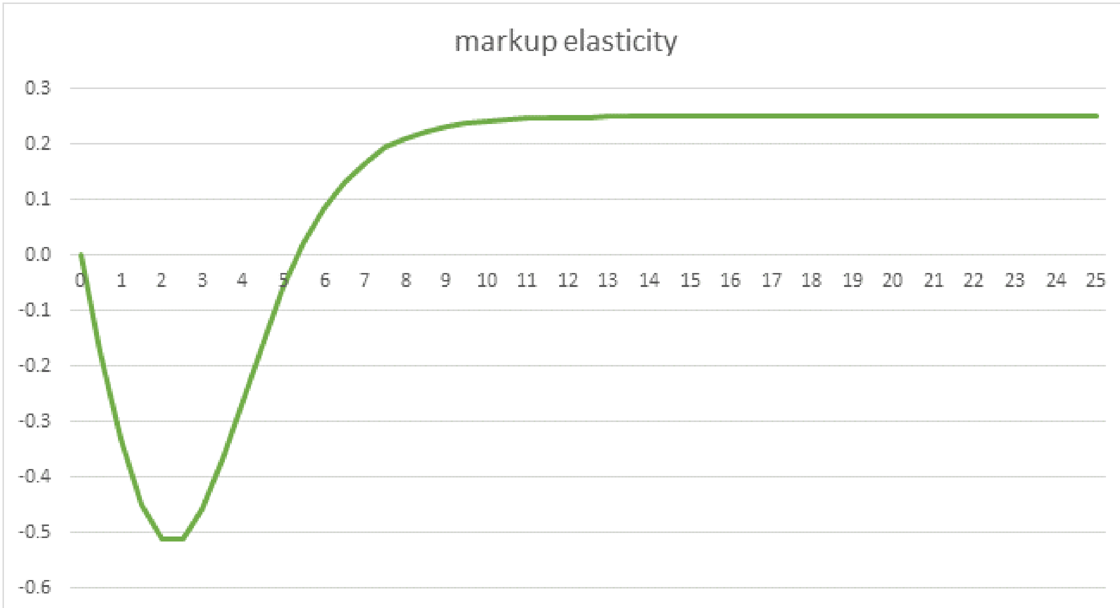

Figure I: The markup elasticity $\varepsilon_{\mu}(\theta)$ as a function of the expected number of firms θ in Example 2.2 where $x_0 = 1$ and $z = 0$. The Pareto distribution of utility shocks has tail index $\lambda = 0.25$.

number of firms. In the limit as $\theta \to 0$, we have $\mu(\theta) \to E_G(x) - z$ and $\varepsilon_{\mu}(\theta) \to 0$. When the number of firms is relatively small, the markup elasticity is negative and the expected markup $\mu(\theta)$ is decreasing. As the number of firms increases, however, the markup elasticity eventually becomes positive and markups are eventually increasing in θ. In the limit as $\theta \to \infty$, we have $\varepsilon_{\mu}(\theta) \to \lambda$, the tail index of G, where $\lambda > 0$.

In this example, we assume that $z = 0$. In general, however, the expected markup is always a non-monotonic function of the expected number of firms whenever the consumer's outside option is strictly less than the minimum firm-specific utility shock, i.e. $z < x_0$. Only in the knife-edge special case where $z = x_0$, i.e. when the consumer's outside option happens to be exactly equal to the minimum utility shock (as in Example 2.1), is the expected markup $\mu(\theta)$ monotonically increasing in the expected number of firms θ.

Figure I provides an illustration of this behavior of the markup elasticity for the case where $\lambda = 0.25$. In this example, competition is price-decreasing when the expected number of firms is less than around five. Eventually, however, when the expected number of firms is greater than around five, competition is price-increasing.

5 Conclusion

This paper examines the effect of competition on markups in random utility models where prices are determined by "limit pricing", i.e. the symmetric equilibrium markup is equal to the difference between the highest and second-highest firm-specific utility shock. In contrast to Gabaix et al. (2016), who focus on environments with a large number of firms, we consider an environment where the number of competing firms is finite but stochastic. The impact of competition on prices depends upon the curvature of $M(\theta)$, the consumer's *expected utility* as a function of the expected number of firms, θ. In particular, competition is price-increasing if and only if the *coefficient of relative risk aversion* of this function, $-M''(\theta)\theta/M'(\theta)$, is strictly less than the elasticity of demand with respect to θ. Whether or not this is true depends not only on the properties of the distribution of utility shocks but also on the expected number of firms and the value of consumer's outside option. For example, when the distribution is Pareto, markups vary *non-monotonically* with the expected number of firms whenever the value of consumer's outside option is strictly less than the minimum utility shock.

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